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Proposed provisions for aseismic design of liquid storage tanks: Part II – Commentary and examples†

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The provisions of IS: 1893-1984 on the aseismic design of liquid storage tanks need revision. Part I of this paper highlighted the necessity and details of the proposed modifications/revisions. In this Part II, a detailed commentary is provided to explain the basis of the proposed codal provisions. Also, two worked out examples are given to illustrate application of these provisions.

Design of liquid storage tanks against earthquake effects is of considerable importance. These structures must remain functional even after an earthquake. In India, the criteria for aseismic design of elevated tower supported tanks are given in IS: 1893-1984[1]. However, no provision is available for aseismic design of ground supported liquid storage tanks. The provisions of IS: 1893-1984 need to be revised to keep abreast of the latest advances in research and design practices. In Part I of this paper, modifications and revisions were proposed for aseismic design provisions of liquid storage tanks given in IS: 1893; This part of the paper provides a detailed commentary explaining the rationale of these modifications. Two worked out examples are included to illustrate application of the proposed revisions. A comparison is also made between the results obtained by applying the proposed revisions and the existing provisions to the same design problem in order to get an idea of the implications of the proposed revisions.

COMMENTARY

Section numbers in this part follow the section numbers of the Part I of the paper. For instance, section C2.1 of this part contains comments on section 2.1 of the Part I. Only those sections of the proposed revisions which require comments are included in the commentary. For clarity, figure numbers 1 to 10 refer to figures in Part I of the paper. The only figure in this part has been numbered as Fig. 11. However, equations and references in this part are unique.

C1: In steel tanks, wall flexibility increases the natural period of the tank so that the resulting response acceleration is significantly higher than the peak ground acceleration. However, in concrete tanks of usual proportions, the increase in natural period due to wall flexibility is not significant enough. Hence, the walls of concrete tanks may be considered as rigid.

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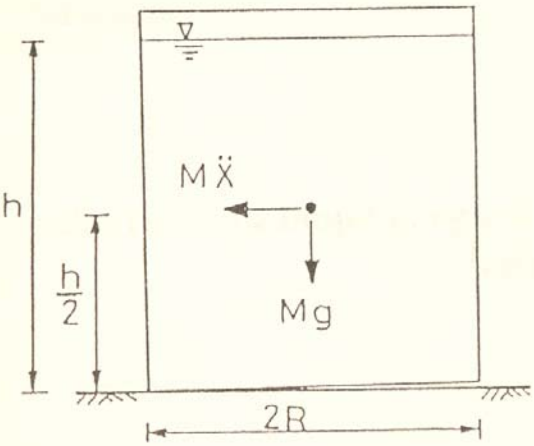

FIG. 11 ANCHORAGE REQUIREMENT FOR TANK

C2: Consider a tank, containing liquid with a free surface, that is subjected to horizontal earthquake ground motion. When the tank walls accelerate, liquid in the lower region of the tank behaves like a mass that is rigidly attached to the tank wall and accelerates along with it. This exerts on the tank walls and base, an impulsive hydrodynamic pressure which is directly proportional to acceleration of the wall. The effect of the impulsive pressure exerted by the wall on the liquid is to excite the liquid into oscillations. This oscillatory motion of the liquid produces convective hydrodynamic pressure on the tank walls and base. Based on this behaviour, Housner[2] proposed a two-mass model for rigid tanks with a free liquid surface and pointed out that an one-mass idealization of the tank is reasonable only for closed tanks that are completely filled with liquid. This viewpoint was also supported by experimental results[3,4] obtained from forced vibration tests of tanks and computational observations. Further, the one-mass idealization of the tank tends to overestimate the contribution of hydrodynamic force towards the design horizontal seismic force. Subsequent theoretical and experimental investigations[5,6] indicated that tank wall flexibility may have a significant influence on hydrodynamic pressure. Hence, Housner's original two-mass model for tanks with rigid walls was suitably modified by Haroun & Housner[5] to incorporate wall flexibility effects in analysis.

C2.1: The various parameters of the two-mass model for a ground supported tank with rigid walls which are given in Fig. 2 are based on Housner[2]. These parameters depend on tank geometry and are obtained by the following expressions:

(a) Circular tank with rigid walls:

Let $\gamma = 1.732\dfrac{R}{h}$ and $\delta = 1.873\dfrac{h}{R}$

$$m_o = \frac{\tanh(\gamma)}{\gamma}\, m$$

$$m_1 = \left[0.455\,\frac{R}{h}\,\tanh(\delta)\right] m$$

$$h_o = \frac{3}{8}\,h \qquad (1)$$

$$h_0' = \frac{3}{8}h\left[1 + \frac{4}{3}\left\{\frac{\gamma}{\tanh(\gamma)} - 1\right\}\right]$$

$$h_1 = h\left[1 - \frac{\cosh(\delta) - 1.0}{\delta\sinh(\delta)}\right]$$

$$h_1' = h\left[1 - \frac{\cosh(\delta) - 2.0}{\delta\sinh(\delta)}\right]$$

$$K_I = 0.836\,\frac{m\,g}{h}\,\tanh^2(\delta)$$

(b) Rectangular tank with rigid walls:

Let $\gamma = 1.732\dfrac{L}{h}$ and $\delta = 1.581\dfrac{h}{L}$

$$m_o = \frac{\tanh(\gamma)}{\gamma}\, m$$

$$m_I = \left[0.527\,\frac{L}{h}\,\tanh(\delta)\right] m \qquad (2)$$

$$h_o = \frac{3}{8}\,h$$

$$h_0' = \frac{3}{8}\,h\left[1 + \frac{4}{3}\left\{\frac{\gamma}{\tanh(\gamma)} - 1\right\}\right]$$

$$h_1 = h\left[1 - \frac{\cosh(\delta) - 1.0}{\delta\sinh(\delta)}\right]$$

$$h_1' = h\left[1 - \frac{\cosh(\delta) - 2.0}{\delta\sinh(\delta)}\right]$$

$$K_I = 0.833\,\frac{m\,g}{h}\,\tanh^2(\delta)$$

Housner has pointed out that for tall tanks with h/L or h/R exceeding 1.5, the expressions for m_o, h_o and h_0' given above, need to be modified. In such tall tanks, the liquid is divided into two parts by a fictitious datum at a depth of 1.5 R or 1.5 L from the liquid surface. The values of m_o, h_o

and h_0' are computed by using the expressions given above for the liquid above this fictitious datum (using h/R or h/L = 1.5). The liquid mass below this datum is considered as a solid (mass $\overline{m}$) which is lumped at its center of gravity. The resultant impulsive mass is the sum of m_o and $\overline{m}$. The heights h_o and h_0' of this resultant impulsive mass above the actual tank bottom are obtained from moment equilibrium.

C2.2: The three-mass model for ground supported circular tank with flexible walls was proposed by Haroun and Housner[5]. Tank wall flexibility does not influence the convective hydrodynamic pressure significantly. Hence, the convective mass, m_l, is assumed to be the same as that for rigid tanks. Tank wall flexibility increases the impulsive mode period significantly. This usually results in an increase in the hydrodynamic force on the tank. The lower impulsive modes of vibration contribute significantly to stresses in the tank wall. The impulsive mass m_r corresponds to the force associated with rigid body motion of the tank wall, while mass m_f corresponds to the force associated with wall deformation relative to ground. The masses m_r and m_f depend on the ratio of tank wall thickness to radius and the ratio of wall to liquid unit weights. However, these parameters are of secondary importance and may be neglected in practical design. The spring-mass model parameters for flexible vertical circular tanks used herein have been adopted from Ref.7.

In the case of flexible rectangular tanks, the impulsive masses m_r and m_f cannot be defined accurately due to lack of analytical solutions. Hence, an approximate solution is obtained by taking m_f equal to m_o, resulting in m_r becoming equal to zero.

C2.3: Most elevated tanks are never completely filled with liquid. Hence, a two-mass idealization of the tank is more appropriate as compared to one-mass idealization.

C3: The response of the two degrees of freedom system can be obtained by elementary structural dynamics. However, for most elevated tanks, it is observed that the two periods are well separated. Hence the system may be conidered as two uncoupled single degree of freedom systems. This method will be satisfactory for design purposes[7], if the ratio of the periods of the two uncoupled systems exceeds 2.5.

C5.1: The expressions for the impulsive period of flexible tanks have been adopted from Ref.7. The expression for a flexible vertical circular tank filled with water was derived for a roofless steel tank with uniform wall thickness. It may also be used for other tank materials and liquids when the mass of tank is relatively small in comparison with that of liquid. The average wall thickness may be used in case of tanks with non-uniform wall thickness. This gives reasonably good approximation for h/R ratio up to 4.0. Thereafter, flexural deformation dominates the response and a more exact period is obtained by using the average wall thickness over the lower half of the tank wall.

For flexible rectangular tanks, the appropriate wall mass should be added to the impulsive mass. In the case of a tank without a roof, the deflection *'d'* may be calculated assuming that the tank wall has a free edge on top and is fixed along the other three edges. The deflection may be obtained from standard handbooks on analysis of plates.

C5.3: The explanatory handbook on codes for earthquake engineering, SP:22 (S & T)-1982[8], and also many text books calculate column lateral stiffness by the expression $12\,E\,I/L^3$. This amounts to considering the girders as infinitely rigid. This results in a significant overestimation of lateral stiffness of staging which reduces the period. Hence, it is important that bracing girder flexibility is also included in analysis.

Simple expressions to calculate the lateral stiffness of moment-resisting frame type tank staging considering the effect of girder flexibility are already available[9,10]. For tank staging with equal panel heights, identical columns arranged along the periphery of a circle, and identical bracing girders, the lateral stiffness of the staging, K_s, may be calculated as:

$$\frac{1}{K_s} = \frac{1}{K_{\text{flexure}}} + \frac{1}{K_{\text{axial}}} \qquad (3)$$

where $$\frac{1}{K_{\text{flexure}}} = \sum_{i=1}^{N_p} \frac{1}{K_{\text{panel}}}$$

$$K_{\text{panel}} = \frac{12\,E\,I_c\,N_c}{h^3}\left[\frac{\frac{I_b}{L}}{\frac{I_b}{L} + \overline{\alpha}\,\frac{I_c}{h}}\right]$$

$\overline{\alpha}$ = 1.0 for upper-most and bottom-most panels
= 2.0 for intermediate panel

$$\frac{1}{K_{axial}} = \frac{2}{N_c A_c E R^2} \sum_{i=1}^{N_p} H_i^2\, h_i$$

C8: Performance factor, *K*, was first introduced in IS:1893-1984 for buildings and not for other structures. This implies that for elevated tanks the value of *K* is 1.0 which is the same as that for buildings with ductile moment resisting frames. This is unreasonable as elevated tank structures have lower energy absorbing capacity and poor ductility as compared to ductile moment resisting frame buildings. Aseismic design codes all over the world prescribe a factor, (similar to the performance factor in IS: 1893-1984) which is 2.8 to 4.5 times higher for elevated tanks than that for buildings with ductile moment resisting frames. This high value of performance factor is justified considering the relatively poor performance of such type of structures during past earthquakes.

Thus, the performance factor (K) is suggested as 3.0 for elevated tanks supported on a staging. In the absence of any investigation on performance factor for ground supported tanks with rigid or flexible walls, a performance factor of 3.0 may be used for them too. It is obvious that a design assuming performance factor, K, of 3.0 will be more expensive. However, the increase in cost is partially offset by savings due to two-mass or three-mass idealization of the tank and consideration of girder flexibility in time period calculation of elevated tanks.

C11: The total hydrodynamic pressure exerted on the tank wall or base consists of two components, namely, the impulsive pressure and the convective pressure. As per IS:1893-1984, the convective pressure can be neglected in the analysis. On the contrary, IS:11682-1985 clearly states that "wherever required the effect of surge due to wave formation of the water may be considered". Convective pressure should be included in the analysis as it may form a significant part of the total hydrodynamic pressure, depending on the tank geometry and proportions[10]. The expressions for estimating the impulsive and convective pressures on the tank wall and base have been adopted from Haroun and Housner[5].

C11.3: The hydrodynamic pressure varies gradually around the circumference of a circular tank. Hence, the stress distribution for maximum pressure intensity (at $\phi = 0$) can be calculated without significant loss of accuracy[11] by assuming the entire tank to be subjected to an axisymmetric pressure distribution equal to the pressure distribution at $\phi = 0$. The resulting axisymmetry significantly reduces the computational effort required.

C11.4: Stress analysis of the tank wall under the simplified linear pressure distribution may be carried out as per IS:3370-Part IV[12] wherein moment and shear coefficients are given for walls of rectangular and circular tanks that are subjected to triangular or rectangular pressure loading.

C12: This condition has been derived by Ishiyama[7]. Consider a tank which is about to rock (Fig. 11). Let M denote the total mass of the tank-liquid system, R, the tank radius, and $\ddot{X}$, the peak response acceleration. Taking moments about the edge,

$$M \ddot{X} \frac{h}{2} = M \, g \, R \tag{4}$$

Taking $\ddot{X}$ as α_{h_f} g for a flexible tank, the condition is

$$\frac{h}{R} = \frac{2}{\alpha_{h_f}} \tag{5}$$

For a rigid tank, α_{h_r} should be used in place of α_{h_f}. Thus, when h/R exceeds the above value, the tank should be anchored to its foundation. The derivation assumes that the entire liquid responds in the impulsive mode. This approximation is reasonable for tanks that are susceptible to overturning with high h/R ratios.

EXAMPLES

Example 1: An elevated water tank of capacity 600 cu.m is located in seismic zone IV. It is circular with an internal diameter 12 m and height 5.6 m and is supported on a R.C. staging 16 m high. The staging consists of 8 columns of 520 mm diameter located on the circumference of a circle of 9m diameter. Horizontal bracing girders of size 200 x 500 mm are provided at a vertical spacing of 4 m. Grade of concrete is M20.

The mass of the empty tank shell is 250 t. The mass of water when the tank is full is 600 t. The total mass of the staging (beams + columns) is 100 t. The height of center of gravity of tank is 2.65 m above top of staging.

Solution 1:

(a) *Two-mass model of tank*

The total liquid mass (m) is 600 t. The depth of water (h) in the tank is 5.3 m. h/R ratio $\equiv$ 0.9. From Fig. 2, $m_0/m = 0.5$, $m_1 / m = 0.475$, $h_o / h = 0.375$. $h_0'/h = 0.9$, $h_1/h = 0.575$, $h_1'/h = 0.825$, and $K_1 h / (mg) = 0.725$. Thus, $m_o = 300$ t, $m_l = 285$ t, $h_o = 1.99$ m, $h_0' = 4.77$ m, $h_l = 3.05$m, $h_1' = 4.37$ m, and $K_l = 805$ kN/m

(b) *Calculation of staging stiffness*

The column and bracing girder properties are:

$E = 2.55 \times 10^4$ N/mm^2

$I_c = 3.59 \times 10^9$ mm^4, $I_b = 2.08 \times 10^9$ mm^4

$h = 4000$ mm, $R = 4500$ mm, $L = 3444$ mm

$I_c / h = 8.98 \times 10^5$ N mm, $I_b / L = 6.04 \times 10^5$ N mm

$12 E I_c N_c / h^3 = 1.37 \times 10^5$ N/mm

$2 / (N_c A_c E R^2) = 2.28 \times 10^{-18}$

For end panels

$K_{panel} = 5.52 \times 10^4$ N/mm ($\overline{\alpha} = 1.0$)

and for the intermediate panels

$K_{panel} = 3.46 \times 10^4$ N/mm ($\overline{\alpha} = 2.0$)

Thus,

$$\frac{1}{K_{flexure}} = 9.40 \times 10^{-5} \text{mm/ N}$$

The center of gravity of the tank is 2.65 m above the top of the staging. Thus, $H_1 = 4650$ mm, $H_2 = 8650$ mm, $H_3 = 12650$ mm, and $H_4 = 16650$ mm. All the panel heights (h_i) are 4000 mm.

Thus

$$\sum_{i=1}^{4} H_i^2 \ h_i = 2.14 \times 10^{12} \text{ mm}^3$$

$$\frac{1}{K_{axial}} = 4.87 \times 10^{-6} \text{ mm/N}$$

$$\frac{1}{K_s} = 9.89 \times 10^{-5} \text{ mm/N}$$

Thus $K_s = 1.01 \times 10^4$ N/mm

(c) *Calculation of time period*
When the tank is empty, the structural mass (m_s) at the center of gravity of the tank is 283 t. When the tank is full, the sum of the structural mass (m_s) and impulsive mass (m_o) is 583 t. Thus, the impulsive mode period for tank empty condition is, $T_o = 1.05$ s

The impulsive mode period for tank full condition is,

$T_o = 1.51$ s

The convective mode period, T_l, is given by

$T_l = 3.74$ s.

(d) *Design horizontal seismic coefficient*
The performance factor, $K = 3.0$; $\beta = 1.0$ (raft foundation on medium soil); $I = 1.5$; and $F_o = 0.25$ (seismic zone IV). When the tank is empty ($T_o = 1.05$ s, damping = 5%), $S_a/g = 0.100$. *When the tank is full* ($T_o = 1.51$ s, damping = 5%), $S_a/g = 0.075$. For convective mode ($T_l = 3.74/s$, damping = 0.5%), $S_a/g = 0.060$. Thus, for tank empty condition, $\alpha_{h_r} = 0.113$; and for tank full condition, $\alpha_{h_r} = 0.084$ and $\alpha_{h_1} = 0.068$.

(e) *Base shear*

Tank empty condition,

$V = V_r = 314$ kN

Tank full condition,

$V_r = 481$ kN

$V_1 = 190$ kN

$$V = \sqrt{V_r^2 + V_1^2} = 517 \text{ kN}$$

Thus tank full condition governs the design. The design lateral force acting at the tank center of gravity is $V = 517$ kN.

(f) *Base moment*

Tank full condition

$h_c = 2.65$m and $h_s = 16.0$m

$M_r = 9490$ kN m

$M_l = 3870$ kN m

The design base moment is $M = 1.02 \times 10^4$ kN m

(g) *Hydrodynamic pressure on tank wall*

(i) Impulsive pressure

The impulsive pressure on the tank wall is given by Eq. (19) of Part I of the paper.

For ease in stress analysis of the tank wall, the pressure variation which occurs at $\cos \phi = 1.0$ (maximum) can be used. Taking $\alpha_{h_r} = 0.084$, $w = 9.81$ kN/m^3, $h =$ 5.3 m, the pressure on the wall is given by

$$p_w = 4.37 \ Q_{iw}(y) \text{ kN/m}^2$$

The value of $Q_{iw}(y)$ for a h/R ratio of 0.833 may be taken the same as that for a h/R ratio of 0.75 (Fig. 8 (a)). Thus, the maximum pressure at the base is 3.71 kN/m^2.

The impulsive pressure on the tank base along a strip of length $2l'$ is given by

$$p_b = 3.78 \sinh(0.327\, x) / \cosh(0.327\, l)$$

For a strip of length $2l'$ that passes through the diameter, $l' = R$.

Thus the impulsive pressure variation along the diameter is given by

$$p_b = 3.78 \ \sinh(0.327\, x) / \cosh(0.327\, R)$$

Similar pressure variation can be obtained along strips parallel to the x–axis.

and for the intermediate panels

$$K_{panel} = 3.46 \times 10^4 \text{ N/mm } (\overline{\alpha} = 2.0)$$

Thus,

$$\frac{1}{K_{flexure}} = 9.40 \times 10^{-5} \text{mm/ N}$$

The center of gravity of the tank is 2.65 m above the top of the staging. Thus, H_1 = 4650 mm, H_2 = 8650 mm, H_3 = 12650 mm, and H_4 = 16650 mm. All the panel heights (h_i) are 4000 mm.

Thus

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When the tank is empty, the structural mass (m_s) at the center of gravity of the tank is 283 t. When the tank is full, the sum of the structural mass (m_s) and impulsive mass (m_0) is 583 t. Thus, the impulsive mode period for tank empty condition is, T_0 = 1.05 s

The impulsive mode period for tank full condition is,

T_o = 1.51 s

The convective mode period, T_l, is given by

T_l = 3.74 s.

(d) *Design horizontal seismic coefficient*

The performance factor, K = 3.0; β = 1.0 (raft foundation on medium soil); I = 1.5; and F_o = 0.25 (seismic zone IV). When the tank is empty (T_o = 1.05 s, damping = 5%), S_a/g = *0.100. When the tank is full* (T_o = 1.51 s, damping = 5%), S_a/g = 0.075. For convective mode (T_l = 3.74/s, damping = 0.5%), S_a/g = 0.060. Thus, for tank empty condition, α_{h_r} = 0.113; and for tank full condition, α_{h_r} = 0.084 and α_{h_1} = 0.068.

(e) *Base shear*

Tank empty condition,

$$V = V_r = 314 \text{ kN}$$

Tank full condition,

$$V_r = 481 \text{ kN}$$

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h_c = 2.65m and h_s = 16.0m

M_r = 9490 kN m

M_l = 3870 kN m

The design base moment is $M = 1.02 \times 10^4$ kN m

(g) *Hydrodynamic pressure on tank wall*

(i) Impulsive pressure

The impulsive pressure on the tank wall is given by Eq. (19) of Part I of the paper.

For ease in stress analysis of the tank wall, the pressure variation which occurs at cos ϕ = 1.0 (maximum) can be used. Taking α_{h_r} = 0.084, w = 9.81 kN/m^3, h = 5.3 m, the pressure on the wall is given by

$$p_w = 4.37 \, Q_{iw} \, (y) \text{ kN/m}^2$$

The value of $Q_{iw}(y)$ for a h/R ratio of 0.833 may be taken the same as that for a h/R ratio of 0.75 (Fig. 8 (a)). Thus, the maximum pressure at the base is 3.71 kN/m^2.

The impulsive pressure on the tank base along a strip of length $2l'$ is given by

$$p_b = 3.78 \sinh(0.327\, x) / \cosh(0.327\, l')$$

For a strip of length $2l'$ that passes through the diameter, $l' = R$.

Thus the impulsive pressure variation along the diameter is given by

$$p_b = 3.78 \sinh(0.327\, x) / \cosh(0.327\, R)$$

Similar pressure variation can be obtained along strips parallel to the x–axis.

(ii) Convective pressure

The maximum displacement of convective mass due to ground excitation,

$\overline{A} = 0.24$ m

$$\theta_h = 1.378 \frac{\overline{A}}{R} \tanh\left(1.837 \frac{h}{R}\right) = 0.05 \text{ rad}$$

The maximum sloshing height = $\theta_h R = 0.05 \text{x} 6.0 = 0.30$ m. The convective pressure on the wall is given by Eq. (25) of Part I of this paper.

Taking $\sin(\omega t) = 1.0$ and $\cos \phi = 1.0$,

$$p_w = 2.67\, Q_{cw}(y)$$

The value of $Q_{cw}(y)$ may be obtained by interpolation. The maximum convective pressure occurs on the liquid surface and is equal to 2.24 kN/m^2.

The convective pressure on the tank bottom is given by Eq. (27) of Part I of this paper and = $4.0\, Q_{cb}(x)$.

The value of $Q_{cb}(x)$ may also be obtained by interpolation. Thus, pressure at the base of the wall is obtained as 0.88 kN/m^2.

The SRSS combination of the maximum impulsive and convective pressure on the tank wall is 3.81 kN/m^2. The maximum hydrostatic pressure on the wall = 52 kN/m^2. Thus, the sum of the hydrodynamic and hydrostatic pressure intensity is 55.8 kN/m^2. Under seismic condition, the permissible stress may be increased by 33%. As 55.8 kN/m^2 is less than 1.33 x 52 kN/m^2, the seismic condition does not govern design of the tank wall.

Solution 2:

In order to compare the proposed provisions with existing practice, the staging of the same tank is analyzed assuming (i) one-mass model of elevated tank, (ii) infinite girder rigidity, and (iii) performance factor $K = 1.0$.

(a) *Calculation of staging stiffness*

Using the panel stiffness based on the lateral stiffness of each column (1.37 x 10^5 N/mm), the stiffness of the staging is $K_s = 3.42 \times 10^4$ N/mm

(b) *Calculation of time period*

When the tank is empty, $T_o = 0.57$ s

When the tank is full, $T_o = 1.00$ s

(c) *Design horizontal seismic coefficient*

$K = 1.0,\ \beta = 1.0,\ I = 1.5$, and $F_o = 0.25$

For tank empty condition ($T_o = 0.57$ s, damping = 5%), $S_a/g = 0.15$ giving $\alpha_h = 0.056$. For tank full condition ($T_0 = 1.00$ s, damping = 5%), $S_a/g = 0.11$ giving $\alpha_h = 0.041$.

(d) *Base shear calculation*

For tank empty condition $V = 156$ kN. For tank full condition, $V = 355$ kN. Therefore, tank full condition is more critical for design and the design base shear is 355 kN.

(e) *Base moment*

The base moment is given by

$M = 6.63 \times 10^3$ kN m

Table 1 compares various quantities obtained by using the proposed provisions and existing practice. It may be noted that though the performance factor, K, is increased from 1.0 to 3.0 (300%), the base shear and base moment increase only by 45% and 55%, respectively. This is due to inclusion of girder flexibility in calculation of staging stiff-

TABLE–1

COMPARISON OF RESULTS OBTAINED BY PROPOSED REVISIONS AND EXISTING PROVISIONS

IDEALIZATION OF TANK	TWO-MASS	ONE-MASS	RATIO
BRACING GIRDER FLEXIBILITY	CONSIDERED	NEGLECTED	
PERFORMANCE FACTOR	$K = 3.0$	$K = 1.0$	
1. Staging Stiffness	1.01x10^4 N/mm	3.42x10^4 N/mm	0.30
2. Time period			
Impulsive			
Tank empty (T_0)	1.05 s	0.57 s	1.84
Tank full (T_0)	1.51 s	1.00 s	1.51
Convective			
Tank full (T_1)	3.74 s	--	--
3. Design horizontal seismic coefficient			
Impulsive			
Tank empty (α_{h_r})	0.113	0.056	2.02
Tank full (α_{h_r})	0.084	0.041	2.05
Convective			
Tank full (α_{h_1})	0.068	--	–
4. Base shear (V)			
Tank empty	314 kN	156 kN	2.02
Tank full	517 kN	355 kN	1.45
5. Base Moment (M)			
Tank full	1.02x10^4 kN m	6.63 x10^3 kN m	1.55

ness and the two-mass idealization of the tank. However, this difference is not consistent and may vary from tank to tank.

Example 2: A ground supported cylindrical steel tank has a radius of 5 m, height 7.5 m and wall thickness 5 mm. The tank is filled with a non-inflammable liquid of specific gravity 1.0. Calculate the seismic force on the tank given that it is in seismic zone V.

Solution:

(a) *Three-mass model of tank*

The total mass of liquid is 589 t. Mass of the tank wall is 9.25 t. The h/R ratio is 1.5. From Fig.2, $m_o/m = 0.71$, $m_l/m = 0.3$, $K_l\, h/(m\, g) = 0.825$, $h_o/h = 0.375$, $h_0'/h = 0.575$, $h_l/h = 0.675$ and $h_1'/h = 0.725$. From Fig. 4, $m_f/m = 0.6$, $h_f/h = 0.46$ and $h_f'/h = 0.58$.

Thus, $m_o = 418$ t, $m_l = 177$ t, $K_l = 636$ kN/m, $h_o = 2.81$ m, $h_0' = 4.31$ m, $h_l = 5.06$m, $h_1' = 5.44$ m, $m_f = 389$ t, $h_f = 3.45$ m and $h_f' = 4.35$ m.

The mass $m_r = 29.5$ t.

(b) *Time period*

(i) Impulsive mode: There are two impulsive mode periods, T_o and T_f. Assuming the foundation as rigid, T_0 is zero. The period coefficient, K_h, is 0.09 for h/R ratio of 1.5 and thickness to radius ratio of 0.001 (Fig. 6). Thus,

$$T_f = 0.10 \text{ s}$$

(ii) Convective mode: The period T_l is given by

$$T_l = 3.31 \text{ s}$$

(c) *Design horizontal seismic coefficient*

Performance factor $K = 3.0$, $\beta = 1.0$, $I = 1.5$, and $F_o = 0.40$. For period T_o (= 0 s) with 2% damping, $S_a/g = 0.10$; for period T_f (= 0.10 s) with 2% damping, $S_a/g = 0.26$; and for period T_l (= 3.31 s) with 0.5% damping, $S_a/g = 0.06$. Thus, $\alpha_{h_r} = 0.180$, $\alpha_{h_f} = 0.468$, and $\alpha_{h_1} = 0.108$.

(d) *Base shear*

$$V_r = 52 \text{ kN}$$

$$V_f = 1830 \text{ kN}$$

$$V_1 = 187 \text{ kN}$$

Design base shear = 1890 kN

(e) *Base moment*

$M_r = -293$ kN m (–ve, neglect this term)

$$M_f = 6320 \text{ kN m}$$

$$M_1 = 947 \text{ kN m}$$

Design base moment = 6390 kN m. This moment is to be used for stress analysis of the tank wall. For calculating the overturning moment for design of the tank foundation, use h_0', h_f' and h_1' in the above expressions. Thus we get, $M_r = 197$ kN m, $M_f = 7920$ kN m and $M_1 = 1020$ kN m. The design overturning moment = 8180 kN m.

(f) *Hydrodynamic pressure on walls*

(i) Impulsive pressure:

The pressure on the wall is given by

$$p_w = 34.4\, Q_{iw}(y) \cos\phi$$

The maximum pressure on the wall occurs at its base with $\cos\phi = 1.0$ and is equal to 24.4 kN/m^2. The pressure on the base is given by

$$p_b = 29.8 \sinh(0.231\, x) / \cosh(0.231 l')$$

The maximum value of p_b occurs at the base of the wall and is equal to 24.4 kN/m^2.

(ii) Convective pressure:

The pressure on the wall (at $\phi = 0$) works out to be

$$p_w = 3.53\, Q_{cw}(y) \sin(\omega t)$$

Thus p_w (max) = 2.98 kN/m^2 on the liquid surface and $p_w = 0.37$ kN/m^2 at the base of the wall. The pressure on the base works out to be

$$p_b = 5.30\, Q_{cb}(x) \sin(\omega t)$$

The maximum value of p_b occurs at the base of the wall and is equal to 0.37 kN/m^2.

Thus, the maximum hydrodynamic pressure which occurs at the base of the wall is equal to 24.4 kN/m^2. The hydrostatic pressure at the wall base is equal to 73.6 kN/m^2. The total pressure intensity at the wall base is equal to 98.0 kN/m^2. Under earthquake condition, 33% increase in permissible stress is allowed. As, 1.33 x 73.6 = 97.9 kN/m^2 is less than the total pressure intensity at the base, the effect of hydrodynamic pressure will govern the design of the tank wall.

The maximum displacement of the convective mass, $\bar{A}$, is 0.29 m. Thus, $\theta_h = 0.08$ radian and the slosh height $(\theta_h R) = 0.4$ m.

(g) *Check for anchorage*
As h/R (1.5) is less than $2/\alpha_{hf}$ (4.27), anchorage of the tank is not essential from overturning consideration.

SUMMARY AND CONCLUSIONS

A detailed commentary explaining basis of the proposed revisions in Part I of the paper on aseismic design of liquid storage tanks is given. Two worked out examples are included to illustrate application of the suggested revisions.

It is noted that consideration of bracing girder flexibility results in a significant reduction in lateral stiffness of the staging. This increases the period significantly and results in a smaller design seismic force. A performance factor (K) of 3.0, proposed in Part I of the paper, will increase the lateral force and base moment. However, this high value of K is justified considering the poor energy absorbing capacity and ductility of tanks. The increase in design force is partially offset by the two or three mass idealization of the tank and by consideration of bracing girder flexibility in elevated tank stagings.

The present code may lead to underdesign of the elevated tanks in the absence of any prescribed value for K. Ignoring of the flexibility of bracing girders for calculation of natural period, coupled with a single mass idealization and absence of K may lead to varying levels of unconservative design for different configurations of the elevated water tanks. These are highlighted by examples.

NOTATION

A_c	Area of cross section of column
$\bar{A}$	Maximum displacement of convective mass
E	Elastic modulus of staging material
E_t	Elastic modulus of tank wall material
F_o	Seismic zone factor
H_i	Height of point of application of lateral force from point of contraflexture in i th panel
I	Importance factor for structure
I_b	Moment of inertia of bracing girder
I_c	Moment of inertia of column
K	Performance factor for tank
K_l	Spring stiffness for convective mode
K_s	Lateral stiffness of staging
L	One-half of tank width; span of bracing girder
M	Design base moment
M_l	Base moment in convective mode
M_r, M_f	Base moment in impulsive mode
N_c	Number of columns
N_p	Number of panels
R	Radius of circular tank; radius of staging
S_a/g	Average acceleration coefficient
T_l	Period of convective mode
T_o	Impulsive mode period for tank with rigid wall
T_f, T_r	Impulsive mode periods for tank with flexible wall
V	Design base shear
V_l	Base shear in Comerclve mode
V_f V_v	Base shear in impulse mode
d	Deflection of the rectangular tank wall due to a uniformly distributed load
g	Acceleration due to gravity
h	Depth of liquid in tank; height of column or panel
h_1, h_1'	Height of convective mass (m_l) above tank base
h_c	Height of center of gravity of tank shell above top of staging
h_f, h_f'	Height of mass m_f above tank base
h_i	Height of columns in i th panel
h_0, h_0'	Height of impulsive mass (m_o) above tank base
h_s	Height of staging
h_t	Height of center of gravity of roof mass above tank base
h_w	Height of center of gravity of wall mass above tank base
m	Total mass of liquid in tank
m_l	Convective mass
m_f, m_r	Impulsive mass for tank with flexible wall
m_o	Impulsive mass for tank with rigid wall
m_s	Mass of tank shell + one-third mass of staging
m_t	Mass of tank roof
m_w	Mass of tank wall
p_w	Liquid Pressure on tank wall
p_b	Liquid Pressure on tank base
w	Unit weight of liquid

α_{h_1} Design horizontal seismic coefficient (= period T_1)

α_{h_f} Design horizontal seismic coefficient (= period T_f)

α_{h_r} Design horizontal seismic coefficient (= period T_o)

β Soil-foundation system coefficient

θ_h Angle of oscillation of liquid surface

ω $2\ \pi/T_1$

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