

Modified proposed provisions for aseismic design of liquid storage tanks: Part II – commentary and examples

O. R Jaiswal* and Sudhir K Jain**

Jain and Medhekar^{3,4} had proposed provisions on aseismic design of liquid storage tanks. In Part I of this paper, need for modification to these provisions was highlighted and a set of modified provisions is proposed. In this part II of the paper, a detailed commentary is provided to explain the rationale of the modified provisions. Two solved numerical examples are also included to illustrate the application of the modified provisions.

Limitations in the provisions of IS 1893:1984¹ on aseismic design of liquid storage tanks have been pointed out by Jain and Sameer², Jain and Medhekar^{3,4} and Rai⁵. Jain and Medhekar^{3,4} have also suggested a set of new provisions which could have been adopted in IS 1893. However, provisions of IS 1893:1984¹ on aseismic design of liquid storage tanks have so far not been revised. Since the work of Jain and Medhekar^{3,4}, considerable amount of new research results on seismic design of liquid storage tanks have been published and most of the international codes have been revised. Moreover, provisions of IS 1893:1984¹ on design horizontal seismic coefficient have also been revised. Considering all these points, in part I of this paper⁶, need for modifications to the provisions suggested by Jain and Medhekar^{3,4} was highlighted and a set of modified provisions are proposed. In the present part, a detailed commentary explaining rationale behind these modified provisions is provided. Two solved numerical examples are also included to illustrate the application of these provisions.

COMMENTARY

Section titles in this paper follow the section titles of Part I of this paper⁶. Commentary has been provided only for those clauses where illustration was needed.

MODIFIED PROVISIONS

Dynamic analysis of liquid containing tank is a complex problem involving fluid-structure interaction. Based on numerous analytical, numerical and experimental studies simple spring mass models of tank-liquid system have been developed to evaluate hydrodynamic forces.

Spring-Mass Model for Seismic Analysis

When a tank containing liquid with a free surface is subjected to horizontal earthquake ground motion, tank wall and liquid are subjected to horizontal acceleration. The liquid in the lower region of tank behaves like a mass that is rigidly connected to tank wall. This mass is termed as impulsive liquid mass, which accelerates along with the wall and induces impulsive hydrodynamic pressure on tank wall and similarly on base. Liquid mass in the upper region of tank undergoes sloshing motion. This mass is termed as convective liquid mass and it exerts convective hydrodynamic pressure on tank wall and base. Thus, total liquid mass gets divided into two parts, i.e., impulsive mass and convective mass. In spring mass model of tank-liquid system, these two liquid masses are to be suitably represented. A qualitative description of impulsive and convective hydrodynamic pressure distribution on tank wall and base is given in Fig. 1.

Sometimes, vertical columns and shaft are present inside the tank. These elements cause obstruction to sloshing motion of liquid. In the presence of such obstructions, impulsive and convective pressure distributions are likely to change. At present, no study is available to quantify effect of such obstructions on impulsive and convective pressures. However, it is reasonable to expect that due to presence of such obstructions, impulsive pressure will increase and convective pressure will decrease.

Ground Supported Tank

The spring mass model for ground supported tank is based on work of Housner⁷. In the spring mass model of tank, h_i is the height at which the resultant of impulsive hydrodynamic pressure on wall is located from the bottom of tank

* Assistant Professor, Department of Applied Mechanics, Visvesvaraya National Institute of Technology, Nagpur 440 011, India.

** Professor, Department of Civil Engineering, Indian Institute of Technology, Kanpur 208 016, India.

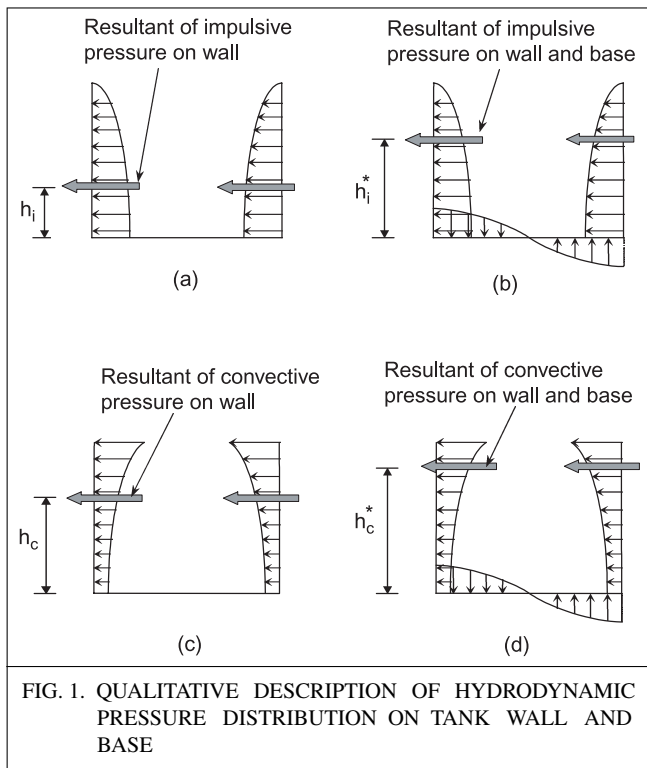


FIG. 1. QUALITATIVE DESCRIPTION OF HYDRODYNAMIC PRESSURE DISTRIBUTION ON TANK WALL AND BASE

wall. On the other hand, h_i^* is the height at which the resultant of impulsive pressure on wall and base is located from the bottom of tank wall. Thus, if effect of base pressure is not considered, impulsive mass of liquid, m_i will act at a height of h_i and if the effect of base pressure is considered, m_i will act at h_i^* . Heights h_i and h_i^* , are schematically described in Figs. 1a and 1b.

Similarly, h_c is the height at which resultant of convective pressure on wall is located from the bottom of tank wall; while, h_c^* is the height at which resultant of convective pressure on wall and base is located. Heights h_c and h_c^* are described in Figs. 1c and 1d.

Circular and rectangular tank

The parameters of spring mass model depend on tank geometry and were originally proposed by Housner⁷. The parameters shown in Figs. 2 and 3 (of part I of the paper⁶) are slightly different from those given by Housner⁷, and have been taken from ACI 350.3⁸. Expressions for these parameters are given in Table 1.

It may be mentioned that these parameters are for tanks with rigid walls. In the literature, spring-mass models for tanks with flexible walls are also available (Haroun and Housner⁹ and Veletsos¹⁰). Generally, concrete tanks are considered as tanks with rigid wall; while steel tanks are considered as tanks with flexible wall. Spring mass models for tanks with flexible walls are more cumbersome to use. Moreover, difference in the parameters (m_i , m_c , h_i , h_i^* , h_c , h_c^* and K_c) obtained from rigid and flexible tank models is not substantial (Jaiswal et al¹¹). Hence in the present code, parameters corresponding to tanks with rigid wall are recommended for all types of tanks.

TABLE 1
EXPRESSIONS FOR PARAMETERS OF SPRING MASS MODEL

Circular Tank	Rectangular Tank
$\frac{m_i}{m} = \frac{\tanh(0.866 \frac{D}{h})}{0.866 \frac{D}{h}}$	$\frac{m_i}{m} = \frac{\tanh(0.866 \frac{L}{h})}{0.866 \frac{L}{h}}$
$\frac{h_i}{h} = 0.375$ for $h/D \leq 0.75$ $= 0.5 - \frac{0.09375}{h/D}$ for $h/D > 0.75$	$\frac{h_i}{h} = 0.375$ for $h/L \leq 0.75$ $= 0.5 - \frac{0.09375}{h/L}$ for $h/L > 0.75$
$\frac{h_i^*}{h} = \frac{0.866 \frac{D}{h}}{2 \tanh(0.866 \frac{D}{h})} - 0.125$ for $h/D \leq 1.33$ $= 0.45$ for $h/D > 1.33$	$\frac{h_i^*}{h} = \frac{0.866 \frac{L}{h}}{2 \tanh(0.866 \frac{L}{h})} - 0.125$ for $h/L \leq 1.33$ $= 0.45$ for $h/L > 1.33$
$\frac{m_c}{m} = 0.23 \frac{\tanh(3.68 \frac{h}{D})}{\frac{h}{D}}$	$\frac{m_c}{m} = 0.264 \frac{\tanh(3.16 \frac{h}{L})}{\frac{h}{L}}$
$\frac{h_c}{h} = 1 - \frac{\cosh(3.68 \frac{h}{D}) - 1.0}{3.68 \frac{h}{D} \sinh(3.68 \frac{h}{D})}$	$\frac{h_c}{h} = 1 - \frac{\cosh(3.16 \frac{h}{L}) - 1.0}{3.16 \frac{h}{L} \sinh(3.16 \frac{h}{L})}$
$\frac{h_c^*}{h} = 1 - \frac{\cosh(3.68 \frac{h}{D}) - 2.01}{3.68 \frac{h}{D} \sinh(3.68 \frac{h}{D})}$	$\frac{h_c^*}{h} = 1 - \frac{\cosh(3.16 \frac{h}{L}) - 2.01}{3.16 \frac{h}{L} \sinh(3.16 \frac{h}{L})}$
$K_c = 0.836 \frac{mg}{h} \tanh^2(3.68 \frac{h}{D})$	$K_c = 0.833 \frac{mg}{h} \tanh^2(3.16 \frac{h}{L})$

Further, flexibility of soil or elastic pads between wall and base do not have appreciable influence on these parameters.

It may also be noted that for certain values of h/D ratio, sum of impulsive mass (m_i) and convective mass (m_c) will not be equal to total mass (m) of liquid; however, the difference is usually small (2 to 3%). This difference is attributed to assumptions and approximations made in the derivation of these quantities.

One should also note that for shallow tanks, values of h_i^* and h_c^* can be greater than h (Figs. 2b and 3b, of part I paper⁶) due to predominant contribution of hydrodynamic pressure on base.

If vertical columns and shaft are present inside the tank, then impulsive and convective masses will change. Though, no study is available to quantify effect of such obstructions, it is reasonable to expect that with the presence of such obstructions, impulsive mass will increase and convective mass will decrease. In absence of more detailed analysis of such tanks, as an approximation, an equivalent cylindrical tank of same height and actual water mass may be considered to obtain impulsive and convective masses.

Elevated tank

- (a) Most elevated tanks are never completely filled with liquid. Hence a two-mass idealization of the tank is more appropriate as compared to a one-mass idealization, which was used in IS 1893:1984¹. Two mass model for elevated tank was proposed by Housner¹²

and is being commonly used in most of the international codes.

Structural mass m_s includes mass of container and one-third mass of staging. Mass of container comprises of mass of roof slab, container wall, gallery, floor slab, and floor beams. Staging acts like a lateral spring and one-third mass of staging is considered based on classical result on effect of spring mass on natural frequency of single degree of freedom system¹³.

Please refer commentary of circular and rectangular tank for effect of obstructions inside the container on impulsive and convective mass.

- (b) The response of the two-degree of freedom system can be obtained by elementary structural dynamics. However, for most elevated tanks it is observed that the two periods are well separated. Hence, the system may be considered as two uncoupled single degree of freedom systems. This method will be satisfactory for design purpose, if the ratio of the period of the two uncoupled systems exceeds 2.5¹⁴.

If impulsive and convective time periods are not well separated, then coupled 2-DOF system will have to be solved using elementary structural dynamics. In this context it shall be noted that due to different damping of impulsive and convective components, this 2-DOF system will have non-proportional damping.

Tanks of Other Shapes

Parameters of spring mass models (i.e., m_i , m_c , h_i , h_i^* , h_c , h_c^* and K_c) are available for circular and rectangular tanks only. For tanks of other shapes, equivalent circular tank is to be considered. Joshi¹⁵ has shown that such an approach gives satisfactory results for intze tanks. Similarly, for tanks of truncated conical shape, Eurocode 8¹⁶ has suggested equivalent circular tank approach.

Time Period of Impulsive Mode

Ground supported circular tank

The coefficient C_i used in the expression of time period T_i and plotted in Fig. 6 (of part I paper⁶), is given by

$$C_i = \left(\frac{1}{\sqrt{h/D} (0.46 - 0.3h/D + 0.067(h/D)^2)} \right)$$

The expression for the impulsive mode time period of circular tank is taken from Eurocode 8¹⁶. Basically this expression was developed for roofless steel tank fixed at base and filled with water. However, this may also be used for other tank materials and fluids. Further, it may be mentioned that this expression is derived based on the assumption that tank mass is quite small compared to mass of fluid. This condition is usually satisfied by most of the tanks. More information on exact expression for time period of circular tank may be obtained from Veletsos¹⁰ and Natchigall et al¹⁷.

In case of tanks with variable wall thickness (particularly, steel tanks with step variation of thickness), thickness of tank wall at one-third height from the base should be used in the expression for impulsive mode time period.

Expression for T_i given in this section is applicable to only those circular tanks in which wall is rigidly attached to base slab. In some concrete tanks, wall is not rigidly attached to the base slab, and flexible pads are used between the wall and the base slab (Figs. 5d to 5f, of part I paper⁶). In such cases, flexibility of pads affects the impulsive mode time period. Various types of flexible connections between wall and base slab described in Fig. 5 (of part I paper⁶) are taken from ACI 350.3⁸, which provides more information on effect of flexible pads on impulsive mode time period.

Ground supported rectangular tank

Eurocode 8¹⁶ and Priestly et al¹⁴ also specify the same expression for obtaining time period of rectangular tank. \bar{h} is the height of combined center of gravity of half impulsive mass of liquid ($m_i/2$), and mass of one wall (\bar{m}_w).

For tanks without roof, deflection, d can be obtained by assuming wall to be free at top and fixed at three edges (Fig. 2a).

ACI 350.3⁸ and NZS 3106¹⁸ have suggested a simpler approach for obtaining deflection, d for tanks without roof. As per this approach, assuming that wall takes pressure q by cantilever action, one can find the deflection, d by considering wall strip of unit width and height, \bar{h} , which is subjected to concentrated load, $P = qh$ (Figs. 2b and 2c). Thus, for a tank with wall of uniform thickness, one can obtain d as follows:

$$d = \frac{P(\bar{h})^3}{3EI_w} \quad (1)$$

$$I_w = \frac{1.0 \times t^3}{12} \quad (2)$$

The above approach will give quite accurate results for tanks with long walls (say, length greater than twice the height). For tanks with roofs and/or tanks in which walls are not very long, the deflection of wall shall be obtained using appropriate method.

Elevated tank

Time period of elevated tank can also be expressed as:

$$T_i = 2\pi \sqrt{\frac{\Delta}{g}}$$

where, Δ is deflection at center of gravity of tank when a lateral force of magnitude $(m_s + m_i)g$ is applied at the center of gravity of tank.

Center of gravity of tank can be approximated as combined center of mass of empty container and impulsive mass of water. The impulsive mass m_i acts at a height of h_i from top of floor slab.

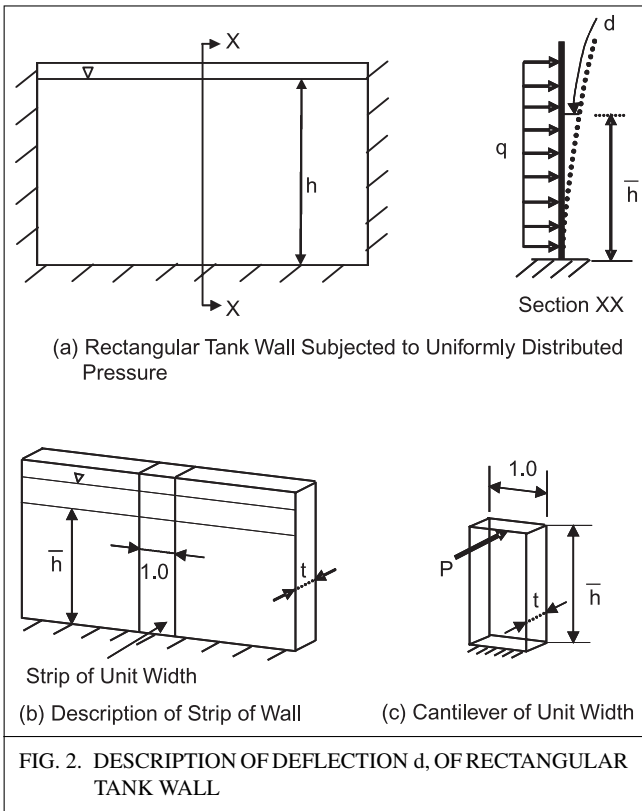


FIG. 2. DESCRIPTION OF DEFLECTION d , OF RECTANGULAR TANK WALL

For elevated tanks with moment resisting type frame staging, the lateral stiffness can be evaluated by computer analysis or by simple procedures¹⁹ or by established structural analysis methods.

In the analysis of staging, due consideration shall be given to modeling of such parts as spiral staircase, which may cause eccentricity in otherwise symmetrical staging configuration.

Time Period of Convective Mode

The expressions for convective mode time period of circular and rectangular tanks are taken from ACI 350.3⁸, which are based on work of Housner⁶. The coefficients C_c in the expressions for convective mode time period plotted in Figs. 6 and 7 (of part I paper⁶) are given below:

(a) For ground supported circular tank:

$$C_c = \frac{2\pi}{\sqrt{3.68 \tanh(3.68h/D)}}$$

(b) For ground supported rectangular tank:

$$C_c = \frac{2\pi}{\sqrt{3.16 \tanh(3.16(h/L))}}$$

Convective mode time period expressions correspond to tanks with rigid wall. It is well established that flexibility of wall, elastic pads, and soil does not affect the convective mode time period.

For rectangular tank, L is the inside length of tank parallel to the direction of loading, as described in Fig. 3.

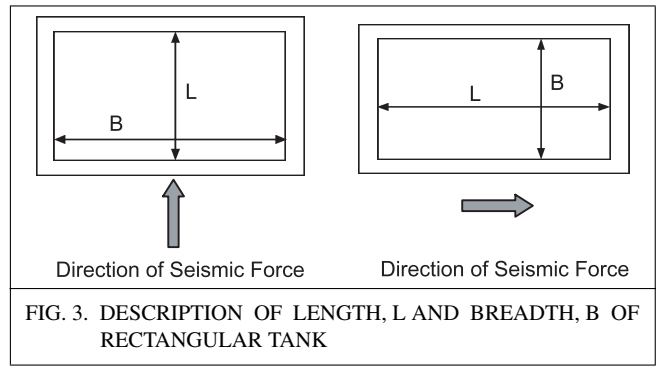


FIG. 3. DESCRIPTION OF LENGTH, L AND BREADTH, B OF RECTANGULAR TANK

Soil Structure Interaction

Soil structure interaction has two effects: Firstly, it elongates the time period of impulsive mode and secondly it increases the total damping of the system. Increase in damping is mainly due to radial damping effect of soil media. A simple but approximate approach to obtain the time period of impulsive mode and damping of tank-soil system is provided by Veletsos¹⁰. This simple approach has been used in Eurocode 8¹⁶ and Priestley et al¹⁴.

Damping

For convective mode damping of 0.5% is used in most of the international codes.

Design Horizontal Seismic Coefficient

Importance factor (I), is meant to ensure a better seismic performance of important and critical tanks. Its value depends on functional need, consequences of failure, and post earthquake utility of the tank.

In this code, liquid containing tanks are put in three categories and importance factor to each category is assigned (Table 1). Highest value of $I = 1.75$ is assigned to tanks used for storing hazardous materials. Since release of these materials can be harmful to human life, the highest value of I is assigned to these tanks. For tanks used in water distribution systems, value of I is kept as 1.5, which is same as value of I assigned to hospital, telephone exchange, and fire station buildings in IS 1893 (Part 1): 2002²⁰. Less important tanks are assigned $I = 1.0$.

Response reduction factor (R), represents ratio of maximum seismic force on a structure during specified ground motion if it were to remain elastic to the design seismic force. Thus, actual seismic forces are reduced by a factor R to obtain design forces. This reduction depends on overstrength, redundancy, and ductility of structure. Generally, liquid containing tanks possess low overstrength, redundancy, and ductility as compared to buildings. In buildings, non structural components substantially contribute to overstrength; in tanks, such non structural components are not present. Buildings with frame type structures have high redundancy; ground supported tanks and elevated tanks with shaft type staging have comparatively low redundancy. Moreover, due to presence of non structural elements like masonry walls, energy absorbing capacity of buildings is

much higher than that of tanks. Based on these considerations, value of R for tanks needs to be lower than that for buildings. All the international codes specify much lower values of R for tanks than those for buildings. As an example, values of R used in IBC 2000²¹ are shown in Table 2. It is seen that for a building with special moment resisting frame value of R is 8.0 whereas, for an elevated tank on frame type staging (i.e., braced legs), value of R is 3.0. Further, it may also be noted that value of R for tanks varies from 3.0 to 1.5.

Type of Structure	R
Building with special reinforced concrete moment resisting concrete frames	8.0
Building with intermediate reinforced concrete moment resisting concrete frames	5.0
Building with ordinary reinforced concrete moment resisting concrete frames	3.0
Building with special steel concentrically braced frames	8.0
Elevated tanks supported on braced/unbraced legs	3.0
Elevated tanks supported on single pedestal	2.0
Tanks supported on structural towers similar to buildings	3.0
Flat bottom ground supported anchored steel tanks	3.0
Flat bottom ground supported unanchored steel tanks	2.5
Reinforced or prestressed concrete tanks with anchored flexible base	3.0
Reinforced or prestressed concrete tanks with reinforced nonsliding base	2.0
Reinforced or prestressed concrete tanks with unanchored and unconstrained flexible base	1.5

Values of R given in the present code (Table 2) are based on studies of Jaiswal et al^{11,22}. In this study, an exhaustive review of response reduction factors used in various international codes is presented. In Table 2, for ground supported and elevated tanks, the highest value of R is 2.5 and lowest value is 1.25. The rationale behind these values of R can be seen from Figs. 4a and 4b.

In Fig. 4a, base shear coefficients (i.e., ratio of lateral design seismic force to weight) obtained from IBC 2000²¹ and IS 1893 (Part 1): 2002²⁰ are compared for a building with special moment resisting frame. This comparison is done for the most severe seismic zone of IBC 2000²¹ and IS 1893 (Part 1): 2002²⁰. It is seen that base shear coefficient from IS 1893 (Part 1): 2002²⁰ and IBC 2000²¹ compare well, particularly up to time period of 1.7 sec.

In Fig. 4b, base shear coefficient for tanks is compared. This comparison is done for the highest as well as lowest value of R from IBC 2000²¹ and present code. It is seen that base shear coefficient match well for highest and lowest value of R . Thus, the specified values of R are quite reasonable and in line with international practices.

Elevated tanks are inverted pendulum type structures and hence, moment resisting frames being used in staging of these tanks are assigned much smaller R values than moment resisting frames of building and industrial frames. For elevated tanks on frame type staging, response

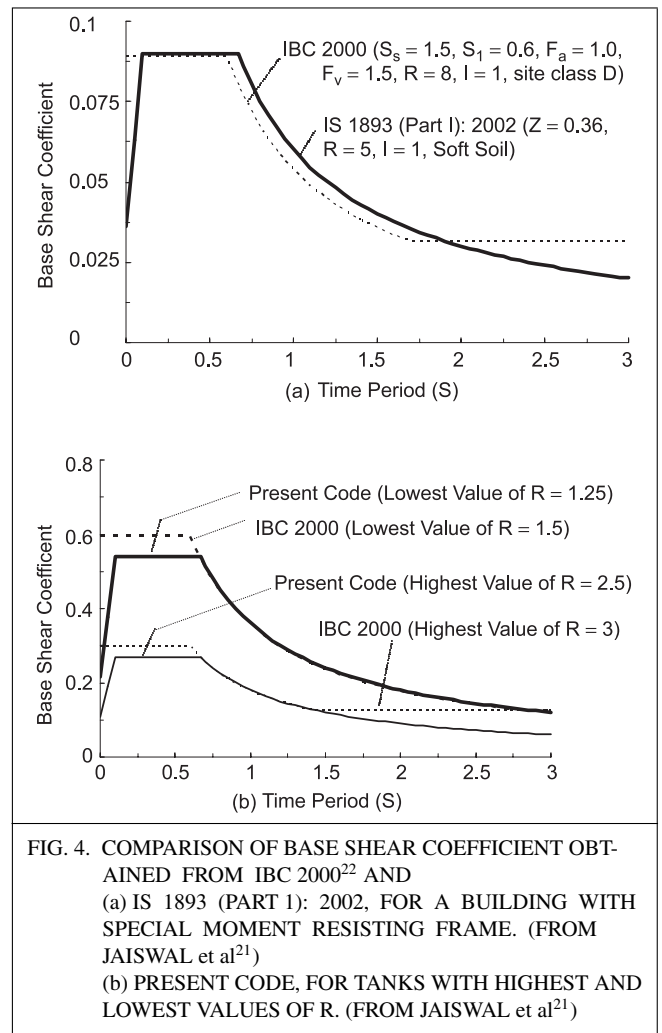


FIG. 4. COMPARISON OF BASE SHEAR COEFFICIENT OBTAINED FROM IBC 2000²² AND (a) IS 1893 (PART 1): 2002, FOR A BUILDING WITH SPECIAL MOMENT RESISTING FRAME. (FROM JAISWAL et al²¹) (b) PRESENT CODE, FOR TANKS WITH HIGHEST AND LOWEST VALUES OF R . (FROM JAISWAL et al²¹)

reduction factor is $R = 2.5$ and for elevated tanks on RC shaft, $R = 1.8$. Lower value of R for RC shaft is due to its low redundancy and poor ductility^{23,24}.

Impulsive and Convective Mode

The values of R given in Table 2 are applicable to design horizontal seismic coefficient of impulsive as well as convective mode.

It may be noted that amongst various international codes, AWWA D-100²⁵, AWWA D-103²⁶ and AWWA D-115²⁷ use same value of R for impulsive and convective modes, whereas, ACI 350.3⁸ and Eurocode 8¹⁶ suggest value of $R = 1$ for convective mode. The issue of value of R for convective component is still being debated by researchers and hence to retain the simplicity in the analysis, in the present provision, same value of R have been proposed for impulsive and convective components.

Average Response Acceleration Coefficient

Response acceleration coefficient (S_a/g) will be given as:

For hard soil sites

$$S_a/g = 2.5 \quad \text{for } T < 0.4$$

$$= 1.0/T \quad \text{for } T \geq 0.4$$

For medium soil sites

$$S_a/g = 2.5 \quad \text{for } T < 0.55$$

$$= 1.36/T \quad \text{for } T \geq 0.55$$

For soft soil sites

$$S_a/g = 2.5 \quad \text{for } T < 0.67$$

$$= 1.67/T \quad \text{for } T \geq 0.67$$

Damping Factor

Table 3 of IS 1893 (Part 1): 2002²⁰ gives values of multiplying factors for 0% and 2% damping, and value for 0.5% damping is not given. One cannot linearly interpolate the values of multiplying factors because acceleration spectrum values vary as a logarithmic function of damping²⁸. In Eurocode 8¹⁶ the value of multiplying factor is taken as 1.673 and as per ACI 350.3⁸ and FEMA 368²⁹, this value is 1.5.

Base Shear

Ground Supported Tank

Live load on roof slab of tank is generally neglected for seismic load computations. However, in some ground supported tanks, roof slab may be used as storage space. In such cases, suitable percentage of live load should be added in the mass of roof slab, m_t .

For concrete or masonry tanks, mass of wall and base slab may be evaluated using wet density of concrete or masonry.

For ground supported tanks, to obtain base shear at the bottom of base slab/plate, shear due to mass of base slab/plate shall be included. If the base shear at the bottom of tank wall is V then, base shear at the bottom of base slab, V' will be given by

$V' = V + (A_h)_i m_b$, where, m_b is mass of base slab or plate.

Elevated Tank

This Clause gives shear at the base of staging. Base shear at the bottom of tank wall can be obtained from Clause of ground supported tank.

Total Base Shear

Except Eurocode 8¹⁶ all international codes use SRSS rule to combine response from impulsive and convective mode. In Eurocode 8¹⁶ absolute summation rule is used, which is based on work of Malhotra et al³⁰. The argument for absolute summation is that the convective mode time period may be several times the impulsive mode period, and hence, peak response of impulsive mode will occur simultaneously when convective mode response is near its peak. However, recently through a numerical simulation for a large number of tanks, Malhotra³¹ showed that SRSS rule gives better results than absolute summation rule.

Base Moment

Ground Supported Tank

- (a) For obtaining bending moment at the bottom of tank wall, effect of hydrodynamic pressure on wall is considered. Hence, m_i and m_c are considered to be located at heights h_i and h_c , which are explained in Figs. 1a and 1c and Clause 3.1.1. Heights, h_i and h_c are measured from top of the base slab or bottom of wall.

Sometimes it may be of interest to obtain bending moment at the intermediate height of tank wall. The bending moment at height, y from bottom will depend only on hydrodynamic pressure and wall mass above that height. Following Malhotra³¹, bending moment at any height y from the bottom of wall will be given by

$$M_i = (A_h)_i [m_i h_i \mu_i + m_w h_w (1 - y/h)^2 / 2 + m_t h_t (1 - y/h)] g$$

$$M_c = (A_h)_c m_c h_c \mu_c g$$

The values of μ_i and μ_c can be obtained from Fig. 5. Second term in the expression of M_i is obtained by considering tank wall of uniform thickness.

- (b) For obtaining overturning moment at the bottom of base slab/plate, hydrodynamic pressure on tank wall as well as tank base is considered. Hence, m_i and m_c are considered to be located at h_i^* , and h_c^* , which are described in Figs. 1b and 1d.

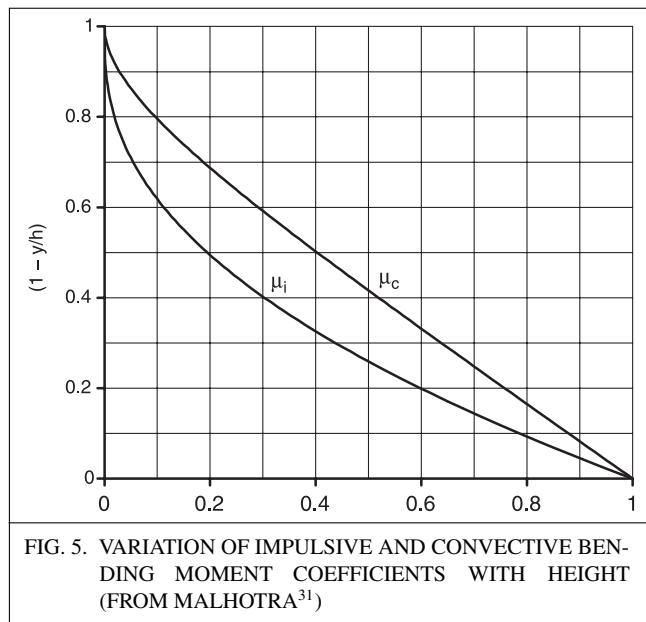


FIG. 5. VARIATION OF IMPULSIVE AND CONVECTIVE BENDING MOMENT COEFFICIENTS WITH HEIGHT (FROM MALHOTRA³¹)

Elevated tank

Structural mass m_s , which includes mass of empty container and one-third mass of staging is considered to be acting at the center of gravity of empty container. Base of staging may be considered at the top of footing.

Total Moment

Refer commentary of Clause on total base shear.

Tank Empty Condition

For tank empty condition, convective mode of vibration will not be generated. Thus, empty elevated tank has to be analyzed as a single degree of freedom system wherein, mass of empty container and one-third mass of staging must be considered.

As such, ground supported tanks shall also be analysed for tank empty condition. However, being very rigid, it is unlikely that tank empty condition will become critical for ground supported tanks.

Direction of Seismic Force

- (a) Base shear and stresses in a particular wall shall be based on the analysis for earthquake loading in the direction perpendicular to that wall.
- (b) For elevated tanks supported on frame type staging, the design of staging members should be for the most critical direction of horizontal base acceleration. For a staging consisting of four columns, horizontal acceleration in diagonal direction (i.e. 45° to X-direction) turns out to be most critical for axial force in columns. For brace beam, most critical direction of loading is along the length of the brace beam.

Sameer and Jain¹⁹ have discussed in detail the critical direction of horizontal base acceleration for some frame staging.

For some typical frame type staging configurations, critical direction of seismic force is described in Fig. 6.

- (c) 100% + 30% rule implies following eight load combinations:

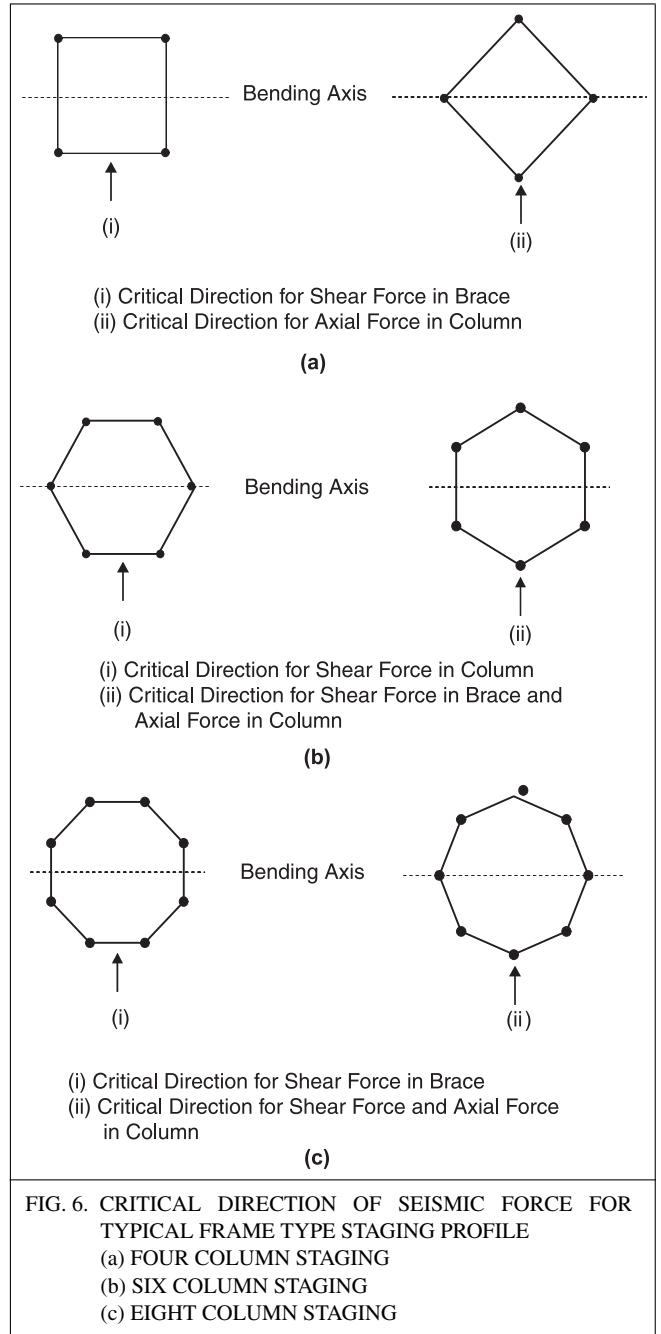
$$\begin{array}{ll}
 (EL_x + 0.3El_y); & (EL_x - 0.3El_y) \\
 -(EL_x + 0.3El_y); & -(EL_x - 0.3El_y) \\
 (0.3EL_x + El_y); & (0.3EL_x - El_y) \\
 -(0.3EL_x + El_y); & -(0.3EL_x - El_y)
 \end{array}$$

Impulsive Hydrodynamic Pressure

The expressions for hydrodynamic pressure on wall and base of circular and rectangular tanks are based on work of Housner⁷.

These expressions are for tanks with rigid walls. Wall flexibility does not affect convective pressure distribution, but can influence on impulsive pressure distribution. This influence also depends on aspect ratio of tanks and for squat tanks this influence is not significant. For a tank with h/D ratio of 2.0, rigid tank model overestimates impulsive pressure at base by about 15%. More details on effect of wall flexibility on impulsive pressure distribution are discussed by Veletsos¹⁰.

Qualitative description of impulsive pressure distribution on wall and base is given in Fig. 1b. Vertical and



horizontal distances, i.e., x, y and circumferential angle, ϕ and strip length l' are described in Fig. 8a (of part I paper⁶).

Convective Hydrodynamic Pressure

The expressions for hydrodynamic pressure on wall and base of circular and rectangular tanks are based on work of Housner⁷.

Qualitative description of convective pressure distribution on wall and base is given in Fig. 1d.

Pressure Distribution in Circumferential Direction

This clause is adapted from Priestley et al¹⁴. Since hydrodynamic pressure varies slowly in the circumferential

direction, the design stresses can be obtained by considering pressure distribution to be uniform along the circumferential direction.

Linearised Pressure Distribution on Wall

Equivalent linear distribution of pressure along wall height is described in Figs. 12b and 12c (of part I paper⁶) respectively, for impulsive and convective pressure.

For circular tanks, maximum hydrodynamic force per unit circumferential length at $\phi = 0$, for impulsive and convective mode, is given by

$$q_i = \frac{(A_h)_i m_i}{\pi D/2} g$$

$$q_c = \frac{(A_h)_c m_c}{\pi D/2} g$$

For rectangular tanks, maximum hydrodynamic force per unit length of wall for impulsive and convective mode is given by

$$q_i = \frac{(A_h)_i m_i}{2B} g$$

$$q_c = \frac{(A_h)_c m_c}{2B} g$$

The equivalent linear pressure distribution for impulsive and convective modes, shown in Figs. 12b and 12c (of part I paper⁶) can be obtained as:

$$a_i = \frac{q_i}{h^2} (4h - 6h_i) \quad b_i = \frac{q_i}{h^2} (6h_i - 2h)$$

$$a_c = \frac{q_c}{h^2} (4h - 6h_c) \quad b_c = \frac{q_c}{h^2} (6h_c - 2h)$$

Pressure due to Wall Inertia

Pressure due to wall inertia will act in the same direction as that of seismic force. For steel tanks, wall inertia may not be significant. However for concrete tanks, wall inertia may be substantial.

Pressure due to wall inertia, which is constant along the wall height for walls of uniform thickness, should be added to impulsive hydrodynamic pressure.

Effect of Vertical Ground Acceleration

Vertical ground acceleration induces hydrodynamic pressure on wall in addition to that due to horizontal ground acceleration. In circular tanks, this pressure is uniformly distributed in the circumferential direction.

Hydrodynamic Pressure

Distribution of hydrodynamic pressure due to vertical ground acceleration is similar to that of hydrostatic pressure. This expression is based on rigid wall assumption. Effect of wall flexibility on hydrodynamic pressure distribution is described in Eurocode 8¹⁶.

Design vertical acceleration spectrum is taken as two-third of design horizontal acceleration spectrum, as per clause 6.4.5 of IS 1893 (Part 1: 2002)²⁰.

To avoid complexities associated with the evaluation of time period of vertical mode, time period of vertical mode is assumed as 0.3 seconds for all types of tanks. However, for ground supported circular tanks, expression for time period of vertical mode of vibration (i.e., breathing mode) can be obtained using expressions given in ACI 350.3⁸ and Eurocode 8¹⁶.

While considering the vertical acceleration, effect of increase in weight density of tank and its content may also be considered.

Sloshing Wave Height

Expression for maximum sloshing wave height is taken from ACI 350.3⁸. Free board to be provided in a tank may be based on maximum value of sloshing wave height. This is particularly important for tanks containing toxic liquids, where loss of liquid needs to be prevented. If sufficient free board is not provided, roof structure should be designed to resist the uplift pressure due to sloshing of liquid.

Moreover, if there is obstruction to free movement of convective mass due to insufficient free board, the amount of liquid in convective mode will also get changed. More information regarding loads on roof structure and revised convective mass can be obtained in Malhotra³¹.

Anchorage Requirement

This condition is described by Priestley et al¹⁴. Consider a tank which is about to rock (Fig. 13, of part I paper⁶). Let M_{tot} denote the total mass of the tank-liquid system, D denote the tank diameter, and $(A_h)_i g$ denote the peak response acceleration. Taking moments about the edge,

$$M_{\text{tot}} (A_h)_i g \frac{h}{2} = M_{\text{tot}} g D/2$$

$$\frac{h}{D} = \frac{1}{(A_h)_i}$$

Thus, when h/D exceeds the value indicated above, the tank should be anchored to its foundation. The derivation assumes that the entire liquid responds in the impulsive mode. This approximation is reasonable for tanks with high h/D ratios that are susceptible to overturning.

Piping

FEMA 368²⁷ provides more information on flexibility requirements of piping system.

Buckling of Shell

More information of buckling of steel tanks is given by Priestley et al¹⁴.

Buried Tanks

The value of response reduction factor for buried tanks is given in Table 2. For buried tanks, the analysis procedure

remains same as that for ground supported tank except for consideration of dynamic earth pressure. For effect of dynamic earth pressure, following comments from Munshi and Sherman³² are taken:

The effect of dynamic earth pressure is commonly approximated by Monobe–Okabe theory^{33,34}. This involves the use of constant horizontal and vertical acceleration from the earthquake acting on the soil mass comprising Coulomb's active or passive wedge. This theory assumes that wall movements are sufficient to fully mobilize the shear resistance along the backfill wedge. In sufficiently rigid tanks (such as concrete tanks), the wall deformation and consequent movement into the surrounding soil is usually small enough that the active or passive soil wedge is not fully activated. For dense, medium-dense, and loose sands, a deformation equal to 0.1, 0.2, and 0.4%, respectively, of wall height is necessary to activate the active soil reaction^{35,36}. Similarly, a deformation of 1, 2, and 4% of the wall height is required to activate the passive resistance of these sands. Therefore, determination of dynamic active and passive pressures may not be necessary when wall deformations are small. Dynamic earth pressure at rest should be included, however, as given by the following equation³³

$$F = k_h \gamma_s H_s^2$$

where k_h is the dynamic coefficient of earth pressure; γ_s is the density of the soil; and H_s is the height of soil being retained. This force acting at height $0.6 h$ above the base should be used to increase or decrease the at-rest pressure when wall deformations are small.

P-Delta Effect

P-delta effect could be significant in elevated tanks with tall staging. P-delta effect can be minimized by restricting total lateral deflection of staging to $h_s/500$, where h_s is height of staging.

For small capacity tanks with tall staging, weight of staging can be considerable compared to total weight of tank. Hence, contribution from higher modes of staging shall also be ascertained. If mass excited in higher modes of staging is significant then these shall be included in the analysis, and response spectrum analysis shall be performed.

EXAMPLES

Example 1: An elevated water tank of capacity 250 m^3 is located in seismic zone IV. Container of tank is of intze type with internal diameter 8.6 m is supported on frame type staging of 16.3 m height. The staging consists of 6 columns of 650 mm diameter located on the circumference of a circle of 6.28 m diameter. Horizontal bracings of size $300 \times 600 \text{ mm}$ are provided at a vertical spacing of 4 m. Grade of concrete is M20.

The mass of the empty tank shell is 157.6 t. The mass of water when the tank is full is 255.7 t. The total mass of the staging (beams + columns) is 103.6 t. The height of center of gravity of empty container is 2.88 m above the top of

circular ring beam. Soil strata is hard. Calculate seismic forces on staging.

Solution: In this example analysis of staging is presented and details about evaluation of hydrodynamic pressure on wall are not included. The next example describes details of hydrodynamic pressure calculation. Tank staging is analyzed using modified provisions given in present paper (Solution 1) and also using provisions of IS 1893:1984 (Solution 2).

Solution 1:

(a) Spring – mass model of tank

The liquid mass (m) is 255.7 t. To obtain the parameters of spring-mass model, equivalent cylindrical container of same volume is considered. The depth of water in the equivalent cylindrical container is 4.4 m. Hence, h/D ratio = 0.51 and from Fig. 2 (of part I paper⁶)

$$m_i/m = 0.55; m_c/m = 0.43; h_i/h = 0.375;$$

$$h_i^*/h = 0.78; h_c/h = 0.61; h_c^*/h = 0.78;$$

$$\text{Thus, } m_i = 140.6 \text{ t}; m_c = 109.9 \text{ t}; h_i = 1.65 \text{ m}; h_i^* = 3.43 \text{ m}; h_c = 2.68 \text{ m}; h_c^* = 3.43 \text{ m}.$$

(b) Calculation of staging stiffness

The staging stiffness is obtained by analyzing finite element model of staging and also by method described by Sameer and Jain¹⁷. Lateral stiffness of staging obtained by finite element software is 17,806 kN/m and that by method of Sameer and Jain¹⁷ is 16,350 kN/m. Stiffness obtained by finite element software is used for further calculations.

(c) Calculation of time period

When the tank is empty, the structural mass (m_s) of the tank is 195.8 t. When the tank is full sum of the structural mass (m_s) and the impulsive mass (m_i) is 336.4 t. Thus, the impulsive mode period for tank empty condition is, $T_i = 0.66 \text{ sec}$. The impulsive mode period for tank full condition is, $T_i = 0.86 \text{ sec}$. The convective mode period is, $T_c = 3.14 \text{ sec}$.

(d) Design horizontal seismic coefficient

The response reduction factor, $R = 2.5$. Importance factor, $I = 1.5$. Zone factor, $Z = 0.24$.

Soil strata is hard.

When the tank is empty ($T_i = 0.66 \text{ sec}$, damping = 5%), $S_a/g = 1.51$

When the tank is full ($T_i = 0.86 \text{ sec}$, damping = 5%), $S_a/g = 1.16$

For convective mode ($T_c = 3.14 \text{ sec}$, damping = 0.5%), $S_a/g = 0.56$.

Thus, for tank empty condition, $(A_h)_i = 0.11$; and for tank full condition, $(A_h)_i = 0.084$ and $(A_h)_c = 0.040$

(e) Base shear

Tank empty condition, $V = V_i = 212 \text{ kN}$.

Tank full condition, $V_i = 277$ kN; $V_c = 43$ kN

$$V = \sqrt{(277)^2 + (43)^2} = 281 \text{ kN}$$

Thus tank full condition governs the design. The design lateral force acting at the center of gravity of tank is $V = 281$ kN.

(f) *Overturing moment*

Tank empty condition, $M^* = 4053$ kN-m.

Tank full condition

$$h_c^* = 3.43 \text{ m} \quad \text{and} \quad h_s = 16.3 \text{ m}$$

$$M_i^* = 5,381 \text{ kN-m}; M_c^* = 852 \text{ kN-m}$$

The total overturning moment is $M^* = 5,448$ kN-m

Solution 2: In order to compare the proposed provisions with existing practice, the staging of the same tank is analyzed assuming (i) one-mass model of elevated tank, (ii) infinite girder rigidity, and (iii) performance factor $K = 1.0$

(a) *Calculation of staging stiffness*

Using the panel stiffness based on the lateral stiffness of each column (36,726 N/mm), the stiffness of staging is $K_s = 55,089$ N/mm

(b) *Calculation of time period*

When the tank is empty, $T_i = 0.37$ sec

When the tank is full, $T_i = 0.57$ sec

(c) *Design horizontal seismic coefficient*

$$K = 1.0, \beta = 1.0, I = 1.5, F_o = 0.25$$

For tank empty condition ($T_i = 0.37$ sec, damping = 5%), $S_a/g = 0.2$ giving $\alpha_h = 0.075$.

For tank full condition ($T_i = 0.57$ sec, damping = 5%), $S_a/g = 0.15$ giving $\alpha_h = 0.056$.

(d) *Base shear calculation*

For tank empty condition, $V = 144$ kN. For tank full condition, $V = 248$ kN.

Therefore, full tank condition is more critical for design and the design base shear is 248 kN.

(e) *Overturing moment*

The overturning moment is given by $M = 4,757$ kN-m

Table 3 Compares various quantities obtained by using the proposed modified provisions and existing practice of IS 1893:1984. It is seen that with the proposed provisions base shear is about 13% higher than that obtained using IS 1893:1984. However, this difference is not consistent and may vary from tank to tank.

Example 2: A ground supported cylindrical steel tank located in seismic zone V, has a diameter of 12 m, height of liquid 8.84 m and wall thickness as 5 mm. Roof of tank consists of stiffened steel plates supported on roof-truss. Tank has a base plate of 10 mm thickness. Tank is rested on hard strata. Calculate the seismic force on the tank.

TABLE 3 COMPARISON OF RESULTS OBTAINED BY MODIFIED PROVISIONS AND IS 1893:1984 ¹			
Idealization of tank	Two-Mass	One-Mass	Ratio
Brace Beam Flexibility	Considered	Neglected	
1. Lateral stiffness of staging	17,806 kN/m	55,089 kN/m	0.32
2. Time period Impulsive mode, Tank empty (T_i) Tank full (T_i) Convective mode, Tank full (T_c)	0.66 sec	0.37 sec	1.78
	0.86 sec	0.57 sec	1.51
	3.14 sec	—	
3. Design seismic horizontal coefficient Impulsive mode Tank empty (A_h) _i Tank full (A_h) _i Convective mode, Tank full (A_h) _c	0.11	0.075	1.47
	0.084	0.056	1.50
	0.040	—	
4. Base shear (V) Tank empty Tank full	212 kN 281 kN	144 kN 248 kN	1.47 1.13
	5. Overturing moment (M^*) Tank empty Tank full	4,053 kN-m 5,448 kN-m	2,762 kN-m 4,757 kN-m

Solution:

(a) *Spring – mass model of tank*

The total liquid mass (m) is 1000t; mass of the tank wall (m_w) is 15.9t; mass of roof (m_r) is 5.1t; h/D ratio = 0.74. From Fig. 2 (of part I paper⁶) $m_i/m = 0.703$; $m_c/m = 0.309$; $h_i/h = 0.375$; $h_i^*/h = 0.587$; $h_c/h = 0.677$; $h_c^*/h = 0.727$. Thus, $m_i = 703$ t; $m_c = 309$ t; $h_i = 3.32$ m; $h_i^* = 5.19$ m; $h_c = 5.98$ m; $h_c^* = 6.43$ m.

(b) *Time period*

The impulsive mode period is, $T_i = 0.13$ sec and convective mode period is, $T_c = 3.64$ sec.

(c) *Design horizontal seismic coefficient*

The response reduction factor, $R = 2.5$. Importance factor, $I = 1.5$. Zone factor, $Z = 0.36$. Soil strata is hard.

For impulsive mode ($T_i = 0.13$ sec, damping = 2%), $S_a/g = 3.5$

For convective mode ($T_c = 3.64$ sec, damping = 0.5%), $S_a/g = 0.48$.

Thus, $(A_h)_i = 0.378$ and $(A_h)_c = 0.052$

(d) *Base shear*

$$V_i = 2,685 \text{ kN}; V_c = 158 \text{ kN}$$

$$V = \sqrt{(2685)^2 + (158)^2} = 2,690 \text{ kN}$$

The design lateral base shear acting at the tank center of gravity is $V = 2,690$ kN.

(e) *Base moment*

Mass of roof, $m_t = 5.1 t$; $h_w = 5.25 m$; $h_t = 10.5025 m$

$M_i = 9, 163 kN\cdot m$; $M_c = 943 kN\cdot m$

The total overturning moment is $M = 9, 211 kN\cdot m$

(f) *Overturning moment*

Mass of base plate, $m_b = 8.9 t$; $h_c^* = 6.43 m$;

$M_i^* = 14, 064 kN\cdot m$; $M_c^* = 1, 015 kN\cdot m$

The total overturning moment is $M^* = 14, 101 kN\cdot m$

(g) *Hydrodynamic pressure on wall*

(i) *Impulsive pressure*

The pressure on the wall is given by

$$p_{iw}(y) = Q_{iw}(y)(A_h)_i \rho g h \cos \phi$$

The maximum pressure on the wall occurs at its base with $\cos \phi = 1.0$ and is equal to $23.60 kN/m^2$.

The pressure on the base slab is given by

$$p_{ib} = 0.866 (A_h)_i \rho g h \frac{\sinh(0.866 \frac{x}{h})}{\cosh(0.866 \frac{h}{h})}$$

The maximum value of p_{ib} occurs at base slab and is equal to $15.0 kN/m^2$.

(ii) *Convective pressure*

The pressure on the wall is given by

$$p_{cw} = Q_{cw}(y)(A_h)_c \rho g h D [1 - 1/3 \cos^2 \Phi] \cos \phi$$

The maximum pressure on the wall occurs at its base with $\cos \phi = 1.0$ and is equal to $0.286 kN/m^2$. Also, at $y = h$, $p_{cw} = 2.31 kN/m^2$

The pressure on the base slab is given by

$$p_{cb} = Q_{cb}(x) (A_h)_c \rho g D$$

$$Q_{cb}(x) = 1.125 \left[\frac{x}{D} - \frac{4}{3} \left(\frac{x}{D} \right)^3 \right] \sec h \left(3.674 \frac{h}{D} \right)$$

The maximum value of p_{cb} occurs at base slab and is equal to $0.303 kN/m^2$.

(iii) *Due to wall inertia*

Pressure on wall due to its inertia is given by,

$$p_{ww} = (A_h)_i t \rho_m g$$

This pressure is uniformly distributed along the wall height and is equal to $0.144 kN/m^2$.

(iv) *Due to vertical excitation*

Hydrodynamic pressure on tank wall due to vertical ground acceleration,

$$p_v = (A_v)[\rho g h (1 - y/h)]$$

where,

$$(A_v) = \frac{2}{3} \left(\frac{Z}{R} \frac{I}{g} \frac{S_a}{g} \right)$$

$(S_a/g) = 1.4 \times 2.5 = 3.5$ for recommended time period of vertical vibration i.e. 0.3 sec corresponding to 2% damping

Hence, $A_v = 0.252$

Thus at $y = 0$, $p_v = 21.87 kN/m^2$

Maximum hydrodynamic pressure is given by

$$p = \sqrt{(p_{iw} + p_{ww})^2 + p_{cw}^2 + p_v^2}$$

Maximum hydrodynamic pressure, which occurs at the base of wall, is $32.28 kN/m^2$. The hydrostatic pressure at the wall base is $86.7 kN/m^2$. The total pressure intensity at the wall base is $118.98 kN/m^2$. Under earthquake condition, 33% increase in permissible stresses is allowed. As, $1.33 \times 86.7 = 115.3 kN/m^2$ is less than the total pressure intensity at the base; the effect of hydrodynamic pressure will govern the design of tank wall.

(h) *Equivalent linear pressure distribution on wall*

The value of equivalent linear impulsive pressure distribution at top and bottom of tank wall is respectively given by $a_i = 27.36 kN/m^2$ and $b_i = 3.96 kN/m^2$. The equivalent linear impulsive pressure distributions are shown in Fig. 7.

The value of equivalent linear convective pressure distribution at top and bottom of tank wall is respectively given by $a_c = -0.052 kN/m^2$ and $b_c = 1.95 kN/m^2$. The equivalent linear convective pressure distributions are shown in Fig. 8.

(i) *Sloshing wave height*

The maximum sloshing wave height of the convective mass is 0.78 m and freeboard provided is 1.18 m.

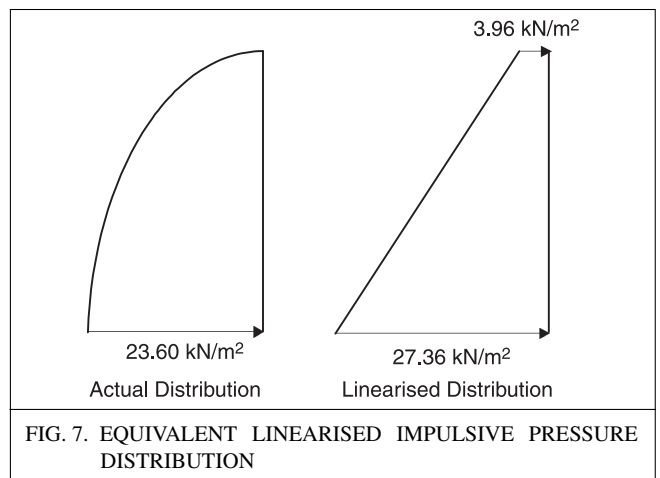


FIG. 7. EQUIVALENT LINEARISED IMPULSIVE PRESSURE DISTRIBUTION

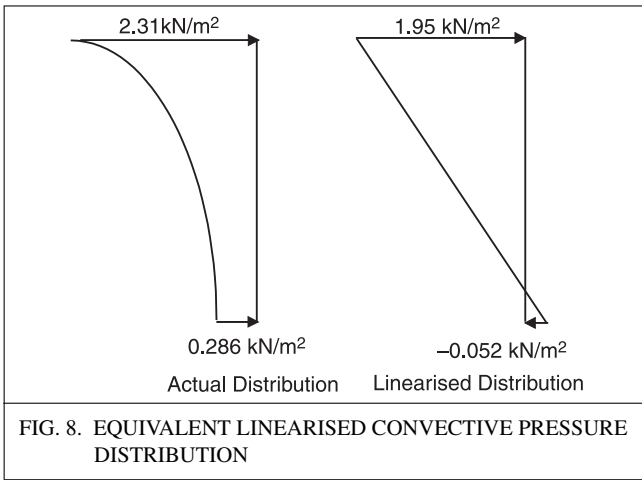


FIG. 8. EQUIVALENT LINEARISED CONVECTIVE PRESSURE DISTRIBUTION

(j) Check for anchorage

As h/D (0.74) is less than $1/(A_h)_i$ i.e. (2.65) anchorage of tank is not essential from overturning consideration.

SUMMARY AND CONCLUSIONS

In this paper detailed commentary describing the rationale of modified provisions given in Part I of this paper is provided. Two solved examples are also included to illustrate the application of these modified provisions.

In modified provisions, values of response reduction factor for different types of tanks have been provided. Generally tanks have low energy absorbing capacity and ductility and hence values of response reduction factors are less than those for buildings with special moment resisting frames. The values of response reduction factors for tanks are arrived on the basis of comparison of design seismic forces with other international codes and hence these values are consistent with present international practices.

Illustrative solved example has clearly shown that for some ground supported tanks, design will be influenced by hydrodynamic forces. This clearly brings out the need for seismic analysis of ground supported tanks, which was not considered in IS 1893:1984. From the solved example on elevated tank it is seen that flexibility of brace beams, which was not considered in, IS 1893:1984¹ and its explanatory handbook (i.e. SP-22³⁷) has significant effect on stiffness of frame type staging. It is seen that provisions of IS 1893:1984¹ can grossly underestimate design forces for certain elevated tanks.

ACKNOWLEDGEMENTS

This work has been supported through a project entitled “Review of Building Codes and Preparation of Commentary and Handbooks” awarded to IIT Kanpur by the Gujrat State Disaster Management Authority (GSDMA), Gandhinagar through World Bank finances. The views and opinions expressed therein are those of the authors and not necessarily those of the GSDMA or the World Bank.

Dr. D. C. Rai, IIT Kanpur; Dr. P. K. Malhotra, FM Global USA; Mr. L. K. Jain, structural designer, Nagpur; Mr. Rushikesh Tirvedi, VMS Consultants, Ahmedabad and Prof. A. R. Chandrasekharan, Hyderabad provided useful comments and suggestions. Mr. Amit Sondeshkar and Ms. Shraddha Kulkarni helped in the preparation of manuscript.

NOTATIONS

- A_v Design vertical seismic coefficient
- $(A_h)_c$ Design horizontal seismic coefficient for convective mode
- $(A_h)_i$ Design horizontal seismic coefficient for impulsive mode
- B Inside width of rectangular tank perpendicular to the direction of seismic force
- C_c Coefficient of time period for convective mode
- C_i Coefficient of time period for impulsive mode
- D Inner diameter of circular tank
- E Modulus of elasticity of tank wall material
- EL_x Response quantity due to earthquake load applied in x -direction
- EL_y Response quantity due to earthquake load applied in y -direction
- I Importance factor given in Table 1 of this code
- I_w Moment of inertia of a strip of unit width of rectangular tank wall
- K_c Spring stiffness of convective mode
- K_s Lateral stiffness of elevated tank staging
- L Inside length of rectangular tank parallel to the direction of seismic force
- M Total bending moment at the bottom of tank wall
- M^* Total overturning moment at base
- M_c Bending moment in convective mode at the bottom of tank wall
- M_c^* Overturning moment in convective mode at the base
- M_i Bending moment in impulsive mode at the bottom of tank wall
- M_i^* Overturning moment in impulsive mode at the base
- M_{tot} Total mass of the tank
- Q_{cb} Coefficient of convective pressure on tank base
- Q_{cw} Coefficient of convective pressure on tank wall
- Q_{ib} Coefficient of impulsive pressure on tank base
- Q_{iw} Coefficient of impulsive pressure on tank wall
- R Response reduction factor given in Table 2 of this paper
- (S_a/g) Average response acceleration coefficient as per IS 1893 (Part 1): 2002 and Clause 3.4 of this paper
- T Time period of tank as defined in IS 1893 (Part 1):2002
- T_c Time period of convective mode (in seconds)
- T_i Time period of impulsive mode (in seconds)
- V Total base shear
- V' Design base shear at the bottom of base slab/plate of ground supported tank

V_c	Base shear in convective mode	q_i	Impulsive hydrodynamic force per unit length of wall
V_i	Base shear in impulsive mode	q_c	Convective hydrodynamic force per unit length of wall
Z	Seismic zone factor as per Table 2 of IS 1893 (Part 1): 2002	t	Thickness of tank wall
a_i, b_i	Values of equivalent linear impulsive pressure distribution on wall at $y = 0$ and $y = h$	t_b	Thickness of base slab
a_c, b_c	Values of equivalent linear convective pressure distribution on wall at $y = 0$ and $y = h$	x	Horizontal distance in the direction of seismic force, of a point on base slab from the reference axis at the center of tank
d	Deflection of wall of rectangular tank, on the vertical center line at a height \bar{h} when loaded by a uniformly distributed pressure q , in the direction of seismic force	y	Vertical distance of a point on tank wall from the bottom of tank wall
d_{\max}	Maximum sloshing wave height	ρ	Mass density of liquid
g	Acceleration due to gravity	ρ_w	Mass density of tank wall
h	Maximum depth of liquid	ϕ	Circumferential angle as described in Fig. 8a
\bar{h}	Height of combined center of gravity of impulsive mass on one wall ($m_i/2$) and mass of wall for rectangular tanks		Deflection of center of gravity of tank when a lateral force of magnitude $(m_s + m_i)g$ is applied at the center of gravity of tank.
h_c	Height of convective mass above bottom of tank wall (without considering base pressure)		
h_i	Height of impulsive mass above bottom of tank wall (without considering base pressure)		
h_s	Structural height of staging, measured from top of footing to the bottom of container wall		
h_t	Height of center of gravity of roof mass above bottom of tank wall		
h_w	Height of center of gravity of wall mass above bottom of tank wall		
h_c^*	Height of convective mass above bottom of tank wall (with considering base pressure)		
h_i^*	Height of impulsive mass above bottom of tank wall (with considering base pressure) for out of plane bending; Refer Clause 3.2.1(b)		
l'	Length of a strip at the base of circular tank, along the direction of seismic force		
m	Total mass of liquid in tank		
m_b	Mass of base slab/plate		
m_c	Convective mass of liquid		
m_i	Impulsive mass of liquid		
m_s	Mass of container of elevated tank and one-third mass of staging		
m_t	Mass of roof slab		
m_w	Mass of tank wall		
\bar{m}_w	Mass of one wall of rectangular tank perpendicular to the direction of loading		
p	Maximum hydrodynamic pressure on wall		
p_{cb}	Convective hydrodynamic pressure on tank base		
p_{cw}	Convective hydrodynamic pressure on tank wall		
p_{ib}	Impulsive hydrodynamic pressure on tank base		
p_{iw}	Impulsive hydrodynamic pressure on tank wall		
p_v	Hydrodynamic pressure on tank wall due to vertical ground acceleration		
p_{ww}	Pressure on wall due to its inertia		
q	Uniformly distributed pressure on one wall of rectangular tank in the direction of ground motion		

REFERENCES

1. IS 1893:1984, "Indian standard criteria for earthquake resistant design of structures", Bureau of Indian standards, New Delhi.
2. Jain, S. K. and Sameer, S. U., "A review of requirements in Indian codes for aseismic design of elevated water tanks", *Bridge and Struct. Engr.*, V. XXIII No. 1, 1993, pp. 1–16.
3. Jain, S. K. and Medhekar, M. S., "Proposed provisions for aseismic design of liquid storage tanks: Part I – Codal provisions", *J. of Struct. Engg.*, V. 20, No. 3, 1993, pp. 119–128.
4. Jain, S. K. and Medhekar, M. S., "Proposed provisions for aseismic design of liquid storage tanks: Part II – Commentary and examples", *J. of Struct. Engg.*, V. 20, No. 4, 1994, pp. 167–175.
5. Rai, D. C., "Review of code design forces for shaft supports of elevated water tanks", *Proc. of 12th Symp. on earthquake Engg.*, Indian institute of technology, Roorkee, India, 2002, pp. 1407–1418.
6. Jain, S. K. and Jaiswal, O. R., "Modified provisions for aseismic design of liquid storage tanks: Part I – Codal provisions", *J. of Struct. Engg.*, V. 32, No. 3, 2005, pp. 195–206.
7. Housner, G. W., "Dynamic analysis of fluids in containers subjected to acceleration", *Nuclear Reactors and Earthquakes, Report No. TID 7024*, U.S. Atomic Energy Commission, Washington D.C. 1963a.
8. ACI 350.3, 2001, "Seismic design of liquid containing concrete structures", American Concrete Institute, Farmington Hill, MI, USA.
9. Haroun, M. A. and Housner, G. W., "Seismic design of liquid storage tanks", *J. of Tech. Councils of ASCE*, V. 107, TC1, 1981, pp. 191–207.
10. Veletsos, A. S., "Seismic response and design of liquid storage tanks", Guidelines for the seismic design

- of oil and gas pipeline systems, Technical Council on Lifeline Earthquake Engineering, ASCE, N.Y., 1984, pp. 255–370.
11. Jaiswal, O. R., Rai, D. C. and Jain, S. K., “Codal provisions on seismic analysis of liquid storage tanks: a review” *Report No. IITK-GSDMA-EQ-04-VI.0*, Indian Institute of Technology Kanpur, Kanpur, 2004b.
 12. Housner, G. W., “The dynamic behavior water tanks”, *Bulletin of Seismological Society of America*, V. 53, No. 2, 1963b, pp. 381–387.
 13. Tse, F. S., Morse, I. E. and Hinkle R. T., “*Mechanical vibration: theory and application*”, 2nd Edition, CBS Publishers and Distributors, 1983, New Delhi.
 14. Priestley, M. J. N., Davidson, B. J., Honey, G. D., Martin, R. J., Ramsay, G., Vessey, J. V. and Wood, J. H., “*Seismic design of storage tanks*”, recommendations of a study group of the New Zealand National society for earthquake Engineering, 1986.
 15. Joshi, S. P., 2000, “Equivalent mechanical model for horizontal vibration of rigid intze tanks”, *ISET J. of earthquake Tech.*, V. 37, No. 1-3, pp. 39–47.
 16. Eurocode 8, 1998, “*Design provisions for earthquake resistance of structures, Part 1- General rules and Part 4 – Silos, tanks and pipelines*”, European Committee for Standardization, Brussels.
 17. Nachtigall, I., Gebbeken, N. and Urrutia-Galicia, J. L., “On the analysis of vertical circular cylindrical tanks under earthquake excitation at its base”, *Engg. Struct.*, V. 25, 2003, pp. 201–213.
 18. NZS 3106, 1986, “*Code of practice for concrete structures for the storage of liquids*”, Standards Association of New Zealand, Wellington.
 19. Sameer, S. U. and Jain, S. K., “Approximate methods for determination of time period of water tank staging”, *The Indian Conc. J.*, V. 66, No. 12, 1992, pp. 691–698.
 20. IS 1893 (Part 1): 2002, “*Indian standard criteria for earthquake resistant design of structures: General provisions and buildings*”, Bureau of Indian standards, New Delhi.
 21. IBC 2000, “*International Building Code International Code Council*”, Falls Church, Virginia, USA, 2000.
 22. Jaiswal, O. R., Rai, D. C. and Jain, S. K., “Codal provisions on design seismic forces for liquid storage tanks: a review”, *Report No. IITK-GSDMA-EQ-01-VI.0*, Indian Institute of Technology Kanpur, Kanpur, 2004a.
 23. Zanh, F. A., Park, R. and Priestley, M. J. N., “Flexural strength and ductility of circular hollow reinforced concrete columns without reinforcement on inside face”, *ACI J.* V. 87, No. 2, 1990, pp. 156–166.
 24. Rai, D. C., “Retrofitting of shaft type staging for elevated tanks”, earthquake spectra, *ERI*, V. 18 No. 4, 2002, pp. 745–760.
 25. AWWA D-100, “*Welded steel tanks for water storage*”, American Water works association, Colorado, USA, 1996.
 26. AWWA D-103, 1997, “*Factory-coated bolted steel tanks for water storage*”, American water works association, Colorado, USA.
 27. AWWA D-115, “*Circular prestressed concrete water tanks with circumferential tendons*”, American Water Works Association, Colorado, USA, 1995.
 28. Newmark, N. M. and Hall, W. J., “*Earthquake spectra and design*”, Engineering monograph, Earthquake Engineering Research Institute, Berkeley, USA, 1982.
 29. FEMA 368, 2000, “*NEHRP recommended provisions for seismic regulations for new buildings and other structures*”, Building Seismic Safety Council, National Institute of Building Sciences, USA.
 30. Malhotra, P. K. Wenk, T. and Wieland, M., “*Simple procedure for seismic analysis of liquid-storage tanks*”, Structural Engineering International, 2000, pp. 197–201.
 31. Malhotra, P. K., “*Seismic analysis of FM approved suction tanks*”, Draft copy, FM Global, USA, 2004.
 32. Munshi, J. A. and Sherman, W. C., “*Reinforced concrete tanks*”, Concrete International, February, 2004, pp. 101–108.
 33. Mononobe, N. and Matsuo, H., “On the determination of earth pressure during earthquakes”, *Proc. of World Engg. Congress*, 1929.
 34. Okabe, S., “General theory of earth pressures”, *J. of the Japanese Soc. of Civ. Engg.*, V. 12, No. 1, 1926.
 35. Ebeling, R. M. and Morrison, E. E. “The seismic design of water front structures”, *NCEL Technical Report, TR-939*, Naval Civil Engineering Laboratory, Port Hueneme, CA, 1993.
 36. Clough, G. W. and Duncan, J. M., “*Chapter 6: Earth pressures*”, Foundation Engineering Handbook, 2nd Edition, NY, 1991, pp. 223–235.
 37. SP-22-1982, “*Explanatory handbook on codes for earthquake engineering*”, Bureau of Indian Standards, New Delhi.