PROPOSED MINIMUM TENSION REINFORCEMENT REQUIREMENT FOR FLEXURAL MEMBERS

by

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ABSTRACT

The provisions regarding minimum tension reinforcement for flexural members in IS:456-1978 and IS:4326-1976 are reviewed. The minimum reinforcement ratio of $0.85/f_y$, as specified by IS:456-1978, is seen to be inadequate for T-beams. The minimum reinforcement provision of IS:4326-1976 may be made the same as that of IS:456. Revised values of minimum tension reinforcement are worked out.

NOTATIONS

A_{st}	-	area of minimum tension reinforcement
b	-	breadth of member
b_f	-	effective breadth of flange of T-beam
b_w	-	breadth of web of T-beam
C	-	compressive force on section
d	-	effective depth of member
D	-	overall depth of member
f_{ck}	-	characteristic strength of concrete cube
f_{cr}	-	modulus of rupture of concrete
f_{cu}	-	average stress in concrete at ultimate strength of section
f_y	-	yield stress of steel
K	-	non-dimensional depth of neutral axis
Ku	-	non-dimensional depth of neutral axis at ultimate strength of section
M	-	moment of resistance of section with minimum tension reinforcement
M_{cr}	-	cracking moment of section
T	-	tensile force on section
εcu	-	crushing strain of concrete
εh	-	strain at which steel begins to strain harden
εsu	-	strain in steel at ultimate strength of section
ρ	-	tension steel ratio
ρ_{min}	-	minimum tension steel ratio

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INTRODUCTION

In the design of reinforced concrete flexural members, it is necessary to provide tension reinforcement greater than a certain minimum amount in order to prevent a sudden failure of the member. Most building codes specify this minimum reinforcement. The requirement of the Indian Standard Code of Practice for Plain and Reinforced Concrete, IS:456-1978[6], stipulates the minimum tension reinforcement ratio for flexural members as $0.85/f_y$; where f_y is the yield stress of steel. This requirement is based on the consideration that tensile capacity of the minimum reinforcement at yield must be at least equal to the tensile force carried by the uncracked concrete. The requirement of the Indian Standard Code of Practice for Earthquake Resistant Design and Construction of Buildings, IS:4326-1976[5], stipulates the minimum tension reinforcement ratio for flexural members as $0.06\, f_{ck}/f_y$ on either face of the member; where f_{ck} is the characteristic strength of concrete cube. This requirement is based on the consideration that tension reinforcement must not strain-harden before the crushin strain of concrete is reached. The argument given for this is that sections having lesser steel "are designated as greatly under-reinforced sections and are not desirable because the reinforcement may fracture before the useful limit of strain of concrete is reached"[3]. The minimum tension steel ratio specified by IS:4326-1976 is greater than that specified by IS:456-1978 for all grades of concrete and steel.

In this paper, the provisions regarding minimum tension reinforcement for flexural members in IS:456-1978 and IS:4326-1976 are reviewed. These are compared with that of the ACI code[1,2]. Revised values of minimum tension reinforcement are suggested for future revisions of IS:456 and IS:4326.

IS:456-1978 PROVISION

IS:456-1978 specifies the minimum tension steel ratio for flexural members as $0.85/f_y$. This provision governs for those beams which, for architectural or other reasons, are much larger in cross section than required by strength considerations alone[8]. Consider a singly reinforced flexural member that is subjected to a gradual increase in load Fig.1. Initially, the member is uncracked and it behaves like a plain concrete beam. As the load is increased, a stage is reached when the extreme concrete tension fiber has tensile stress equal to the modulus of rupture of concrete (f_{cr}). At this stage, cracks begin to form in the tension zone of the member; the corresponding value of moment is the cracking moment of the section, M_{cr}. A further increase in load causes all tension on the section to be taken by the tension reinforcement as the tension concrete is cracked and hence cannot carry any tensile force. Thus, the tensile capacity of the tension reinforcement must be greater than the tensile force carried by the uncracked concrete so that the section can resist the cracking moment in a ductile manner. This will prevent sudden failure of the flexural member when the cracking moment is reached. Thus, the minimum tension steel ratio, 'p_{min}' can be found by equating the cracking moment of the section, M_{cr}, to the strength computed as a reinforced concrete section, M and solving for 'p_{min}'. For a rectangular section, the cracking moment is given by

$$M_{cr} = f_{cr}\, b\, D^2/6 \qquad ...(1)$$

where the modulus of rupture of concrete, f_{cr}, is taken as $0.7 \sqrt{f_{ck}}$[6]; b = breadth of member and D = overall depth of member. The moment of resistance of a rectangular singly reinforced concrete section with minimum reinforcement is given by

$$M = 0.87\, f_y\, \rho_{min}\, (1.0 - 0.416\, K)\, b\, d^2 \qquad ...(2)$$

where d = effective depth of member; 'ρ_{min}' = A_{st}/bd; A_{st} = area of minimum tension reinforcement and K = non=dimensional depth of the neutral axis, which is given by

$$K = \frac{0.87\, f_y\, \rho_{min}}{0.36\, f_{ck}} \qquad ...(3)$$

From equations 1, 2 and 3, the minimum tension steel ratio, 'ρ_{min}' works out as

$$\rho_{min} = \frac{f_{ck}}{2\, f_y} \left\{ 1.0 - \sqrt{1.0 - \frac{0.536}{\sqrt{f_{ck}}} \left\{ \frac{D}{d} \right\}^2} \right\} \qquad ...(4)$$

For reinforced concrete beams, the ratio of effective depth to overall depth may be about 0.9. Using this value, Table 1 gives the values of 'ρ_{min}' as per equation 4. These values may be approximated as ρ_{min}' = 0.173 $\sqrt{f_{ck}} / f_y$. The constant 0.173 may be rounded off and thus, ρ_{min} for a rectangular section may be expressed as

$$\rho_{min} = \frac{0.18\sqrt{f_{ck}}}{f_y} \qquad ...(5)$$

Table 1 also gives the values of ρ_{min} as per equation 5. It is seen that the existing minimum of $0.85/f_y$ is adequate for rectangular beams of concrete grade up to M25 only.

However, beams in reinforced concrete construction are often cast monolithic with the slab. These beams behave like T-beams. A similar analysis must also be carried out assuming the flange of the T-beam in (i) compression and (ii) tension.

A parametric study was carried out to determine ρ_{min} (=$A_{st}/b_w d$) for T-beams of usual proportions, with the flange in compression and in tension. The ratio of effective breadth of flange (b_f) to breadth of web (b_w) was varied from 4 to 6; the ratio of overall depth (D) to depth of flange (d_f) was varied from 3 to 6 and the ratio of effective depth (d) to overall depth (D) was taken as 0.9 Fig. 2. Table 2 gives the values of ρ_{min} obtained with the flange of the T-beam in compression. These values are significantly higher than the existing minimum of $0.85/f_y$. Table 3 gives the maximum values of ρ_{min} obtained in Table 2 (for $D/d_f = 6$ and $b_f/b_w = 6$). These values may be obtained by the approximate relation

$$\rho_{min} = \frac{0.24\sqrt{f_{ck}}}{f_y} \qquad ...(6)$$

Table 3 also gives the values of ρ_{min} as per equation 6.

Table 4 gives the values of ρ_{min} obtained with the flange of the T-beam in tension. This case is applicable for simply supported and continuous beams which project above the slab and for cantilever T-beams with the flange in tension. The first row of Table 5 gives the maximum values of ρ_{min} obtained in Table 4 (for $D/d_f = 3$ and $b_f/b_w = 6$). However, some minimum reinforcement will always be provided in that part of the slab (edge strip) which acts integrally with the beam. This minimum reinforcement is 0.15 percent and 0.12 percent of the concrete section for steel of grade Fe 250 and Fe 415 respectively. Assume that this reinforcement is also effective in resisting the bending moment. The minimum reinforcement required in the web of the T-beam can be found by deducting the minimum reinforcement in the flange from the total minimum tension reinforcement required for the T-beam. The second row of Table 5 gives the minimum tension reinforcement thus obtained. These values are also significantly higher than the existing minimum of $0.85/f_y$. Minimum tension reinforcement for the web of T-beams with the flange in tension be obtained by the approximate relation,

$$p_{min} = \frac{0.53\sqrt{f_{ck}}}{f_y} \qquad \text{for } f_y = 250 \qquad \text{...(7a)}$$

$$p_{min} = \frac{0.48\sqrt{f_{ck}}}{f_y} \qquad \text{for } f_y = 415 \qquad \text{...(7b)}$$

The third row of Table 5 gives the values of p_{min} as per equation 7a and b.

IS:4326-1976 PROVISION

The minimum tension steel ratio of $0.06\ f_{ck}/f_y$ as specified by IS:4326-1976 is based on the consideration that the tension steel must not strain-harden at the ultimate strength of the section. This is to prevent fracture of reinforcement before the crushing strain of concrete is reached. Consider a singly reinforced section at its ultimate strength Fig.3. From the strain diagram, the non-dimensional depth of the neutral axis at the ultimate strength of the section Ku is given by

$$Ku = \frac{\varepsilon_{cu}}{E_{cu} + E_{su}} \qquad \text{...(8)}$$

where ε_{cu} is the crushing strain of concrete and ε_{su} is the strain in the tension steel. SP:22[7] limits the value of ε_{su} to εh which is the strain at which the tension steel begins to strain harden. Equilibrium of the section, C = T, gives

$$Ku = \frac{P\,f_y}{f_{cu}} \qquad \text{...(9)}$$

where f_{cu} is the average stress is concrete at the ultimate strength of the section. Taking $\varepsilon_{su} = \varepsilon h$ in equation 8.

equations 8 and 9 give

$$P = \left\{\frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon h}\right\} \frac{f_{cu}}{f_y} \qquad \text{...(10)}$$

SP:22[7] uses equation 10 to specify the minimum tension reinforcement for flexural members in IS:4326-1976. However, it is not appropriate to limit the strain in the tension steel to εh at the ultimate strength of the section. This is because no immediate failure results if the steel strain exceeds εh. For mild steel, the steel strain at fracture of reinforcement is almost 10 times εh. Hence, there is no danger of fracture of reinforcement. Further, this type of failure will be ductile Fig.4. In contrast, the criterion used by IS:456-1978 ensures against a sudden and brittle failure which will result if the tensile force carried by the uncracked concrete is greater than the tensile capacity of the tension steel Fig.1. The criterion used by SP:22 and IS:4326-1976 to determine the minimum reinforcement is neither used by the ACI code[1,2] nor used by Blume, et al.[3]. Seismic loads do not put any additional criteria for determining the minimum tension reinforcement. Thus, there is no need for a provision in IS:4326 which is different from that in IS:456.

ACI CODE PROVISION

The ACI code[1,2] uses the same criterion as used by SP:24[8] for determining the minimum tension reinforcement ratio for flexural members and specifies it as $1.38/f_y$. This minimum reinforcement must be provided wherever positive reinforcement is needed, except where both positive and negative reinforcement are one-third greater than required by analysis. This exception provides sufficient additional reinforcement in large members where $1.38\ bd/f_y$ would be excessive. Wang and Salmon[9] point out that this value is "reasonable for isolated rectangular beams and the usual positive moment orientation for T-sections; it is unconservative for T-sections with the slab in tension." Ferguson, et al.[4] mention that "the ACI code committee is working on new expressions that are more sensitive to member shape and a range of material strengths."

SUMMARY AND CONCLUSIONS

A minimum tension steel ratio of $0.85/f_y$ is specified by IS:456-1978 for flexural members so as to prevent a sudden failure which may result if tensile capacity of tension steel is less than the tension carried by uncracked concrete. This criterion is used to evaluate the minimum tension steel ratio for rectangular and T-beams. The existing minimum of $0.85/f_y$ is found to be adequate for rectangular beams only. It is significantly lower than the minimum required for T-beams. Simple expressions are proposed for the minimum tension steel ratio for rectangular beams and for T-beam when the flange is in compression or tension. The proposed minimum steel requirement is higher than the existing minimum of $0.85/f_y$ for rectangular beams of concrete grade M25 and above and for T-beams for all grades of concrete. The minimum reinforcement ratio of $0.06\ f_{ck}/f_y$ as specified by IS:4326-1976, is based on the criterion that the tension steel must not strain-harden at the ultimate strength of the section. However, this criterion is inappropriate and is not used by seismic codes elsewhere. Seismic loads do not impose any additional criteria for determining the minimum tension reinforcement. Thus, there is no need for a separate provision in IS:4326; it should be the same as that specified by IS:456.

It is recommended that the minimum tension reinforcement for flexural members, 'ρ_{min}' in both IS:456 and IS:4326, be taken as:

(i) $\rho_{min} = 0.18\ \sqrt{f_{ck}/f_y}$ for rectangular beams

(ii) $\rho_{min} = 0.24\ \sqrt{f_{ck}/f_y}$ for T-beams with the flange in compression

(iii) $\rho_{min} = 0.53\ \sqrt{f_{ck}/f_y}$ for the web of T-beams with flange in tension ($f_y = 250$)

(iv) $\rho_{min} = 0.48\ \sqrt{f_{ck}/f_y}$ for the web of T-beams with flange in tension ($f_y = 415$)

These requirements may be relaxed where the area of reinforcement provided is at least one half greater than that required by analysis. This may be allowed since "extra safety is a satisfactory substitute for having a ductile failure mode"[9].

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Table 1: Minimum Tension Reinforcement for Rectangular Sections

f_{ck} (MPa)	15	20	25	30	35	40
$\rho_{min} f_y$ as per eq. (4)	0.67	0.77	0.86	0.94	1.01	1.08
$\rho_{min} f_y$ as per eq. (5)	0.70	0.80	0.90	0.99	1.06	1.14

Table 2: Minimum Tension Reinforcement for T-beams with the Flange in Compression (d/D = 0.9)

Concrete Grade	D / df	bf / bw = 4	bf / bw = 5	bf / bw = 6
M 15	3	0.868	0.898	0.923
	4	0.872	0.900	0.922
	5	0.873	0.903	0.926
	6	0.870	0.901	0.926
M 20	3	1.000	1.035	1.064
	4	1.004	1.037	1.063
	5	1.006	1.041	1.067
	6	1.002	1.039	1.067
M 25	3	1.116	1.156	1.189
	4	1.121	1.158	1.187
	5	1.123	1.162	1.192
	6	1.119	1.161	1.192
M 30	3	1.222	1.265	1.301
	4	1.227	1.268	1.300
	5	1.229	1.272	1.305
	6	1.225	1.270	1.305
M 35	3	1.318	1.365	1.405
	4	1.325	1.369	1.403
	5	1.327	1.373	1.409
	6	1.322	1.371	1.409
M 40	3	1.409	1.459	1.501
	4	1.415	1.462	1.499
	5	1.417	1.467	1.505
	6	1.412	1.465	1.506

Table 3: Minimum Tension Reinforcement for T-beams with the Flange in Compression ($d/D = 0.9$, $D/d_f = 6$ and $b_f/b_w = 6$)

f_{ck} (MPa)	15	20	25	30	35	40
$\rho_{min} f_y$ from Table 2	0.93	1.07	1.20	1.31	1.41	1.51
$\rho_{min} f_y$ as per eq. (6)	0.93	1.07	1.20	1.31	1.42	1.52

Table 4: Minimum Tension Reinforcement for T-beams with the Flange in Tension (d/D = 0.9)

Concrete Grade	D / d_f	$b_f / b_w = 4$	$b_f / b_w = 5$	$b_f / b_w = 6$
M 15	3	1.968	2.345	2.707
	4	1.918	2.312	2.702
	5	1.818	2.198	2.582
	6	1.712	2.066	2.426
M 20	3	2.221	2.630	3.017
	4	2.166	2.595	3.012
	5	2.056	2.471	2.884
	6	1.939	2.328	2.717
M 25	3	2.447	2.888	3.300
	4	2.387	2.850	3.294
	5	2.268	2.717	3.158
	6	2.141	2.562	2.981
M 30	3	2.653	3.123	3.560
	4	2.589	3.082	3.554
	5	2.461	2.941	3.410
	6	2.325	2.776	3.222
M 35	3	2.843	3.341	3.802
	4	2.775	3.298	3.796
	5	2.639	3.149	3.644
	6	2.495	2.973	3.445
M 40	3	3.020	3.545	4.029
	4	2.949	3.500	4.023
	5	2.805	3.343	3.863
	6	2.653	3.158	3.655

Table 5: Minimum Tension Reinforcement for T-beams with the Flange in Tension ($d/D = 0.9$, $D/d_f = 3$ and $b_f/b_w = 6$)

Row No.	f_{ck} (MPa)		15	20	25	30	35	40
1	$\rho_{min} f_y$ (flange + web)*		2.71	3.02	3.30	3.56	3.80	4.03
2	$\rho_{min} f_y$ (web)**	$f_y = 250$	2.01	2.32	2.61	2.87	3.11	3.34
		$f_y = 415$	1.79	2.10	2.38	2.64	2.88	3.11
3	$\rho_{min} f_y$ (web** eq. 7a and b	$f_y = 250$	2.05	2.37	2.65	2.90	3.14	3.35
		$f_y = 415$	1.86	2.15	2.40	2.63	2.84	3.04

* - required for the flange and web of T-beam.

** - required for the web of T-beam (assuming minimum reinforcement for slabs is provided in flange).

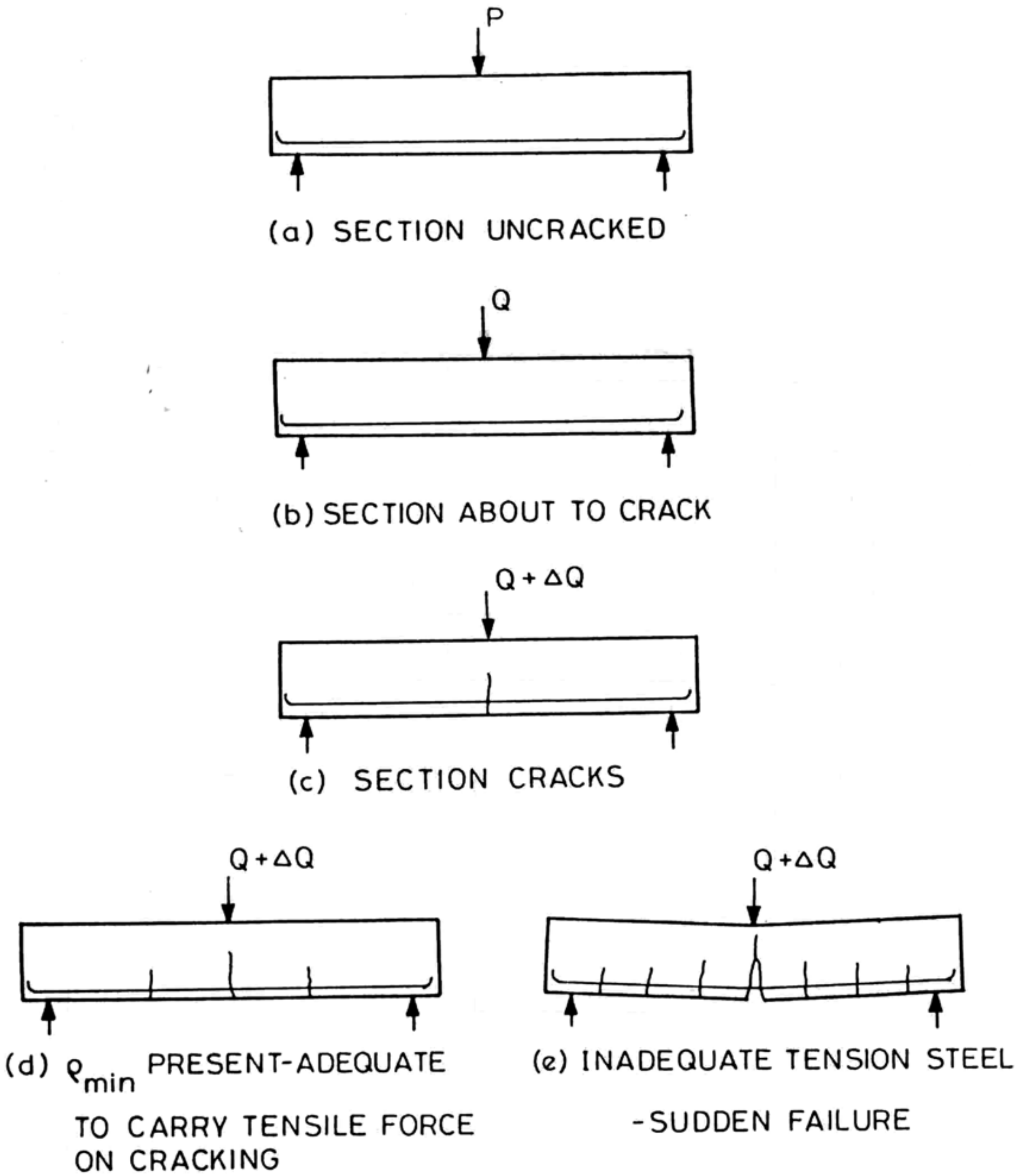
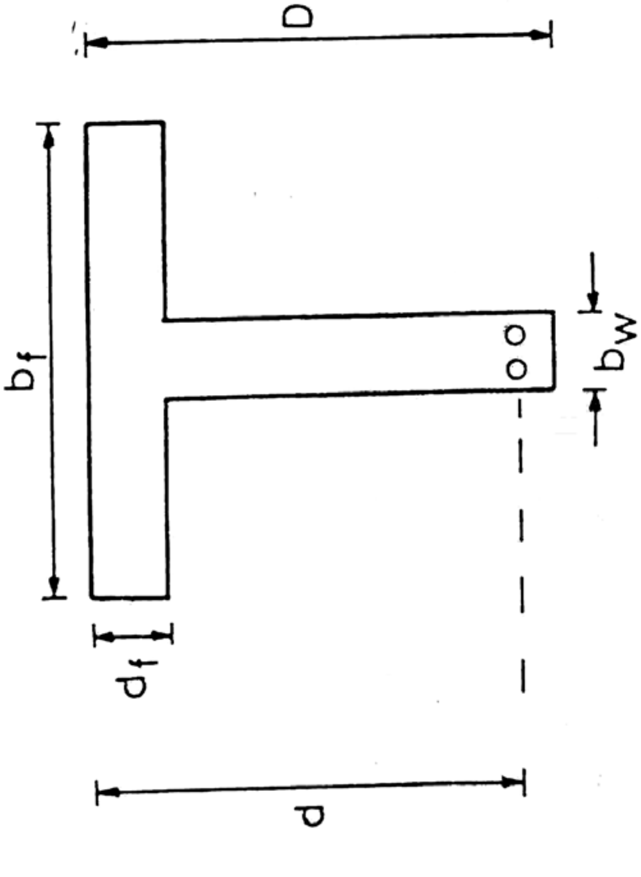


Fig. 1 : Criterion for Determining Minimum Tension Reinforcement-IS: 456-1978



$$\frac{d}{D} = 0.9\ ; \quad \frac{d_f}{D} = \frac{1}{3} \sim \frac{1}{6}\ ; \quad \frac{b_f}{b_w} = 4 \sim 6$$

FIG.2: T-BEAM-NOMENCLATURE AND PROPORTIONS

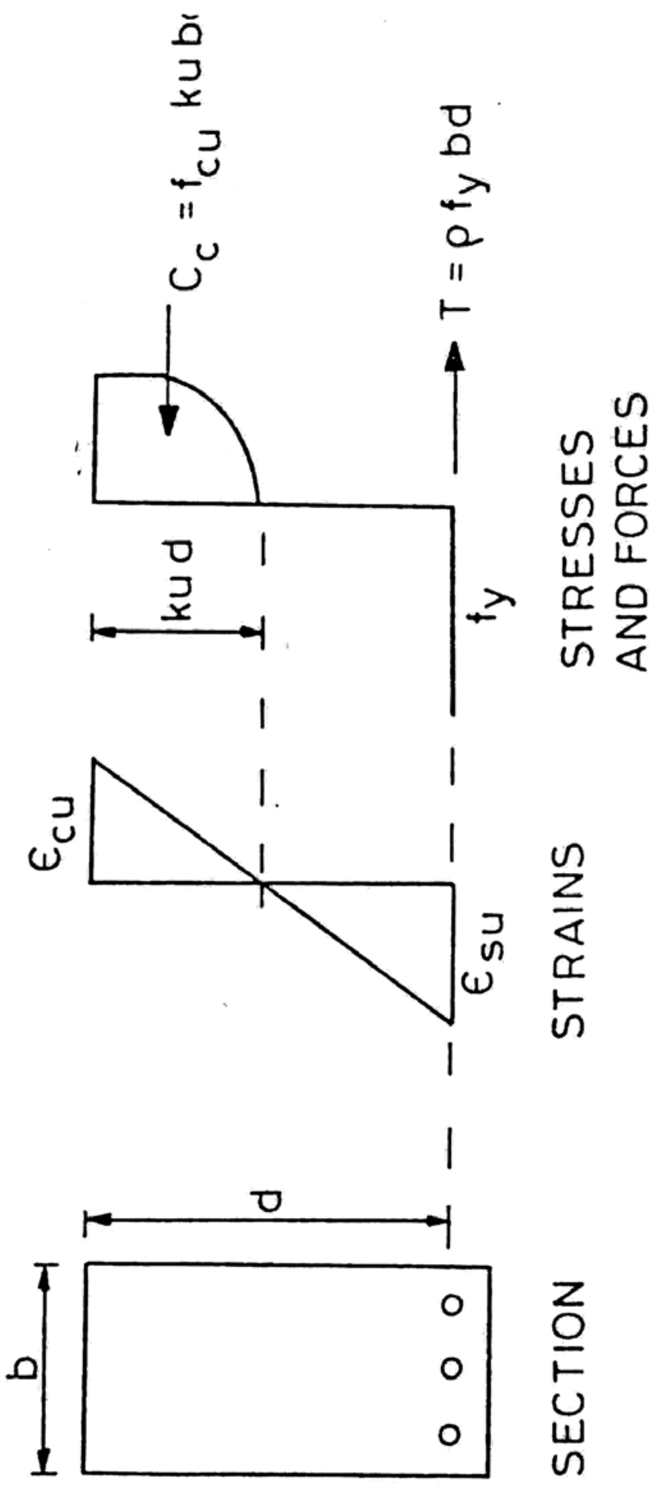


FIG.3: A SINGLY REINFORCED SECTION AT ITS ULTIMATE STRENGTH

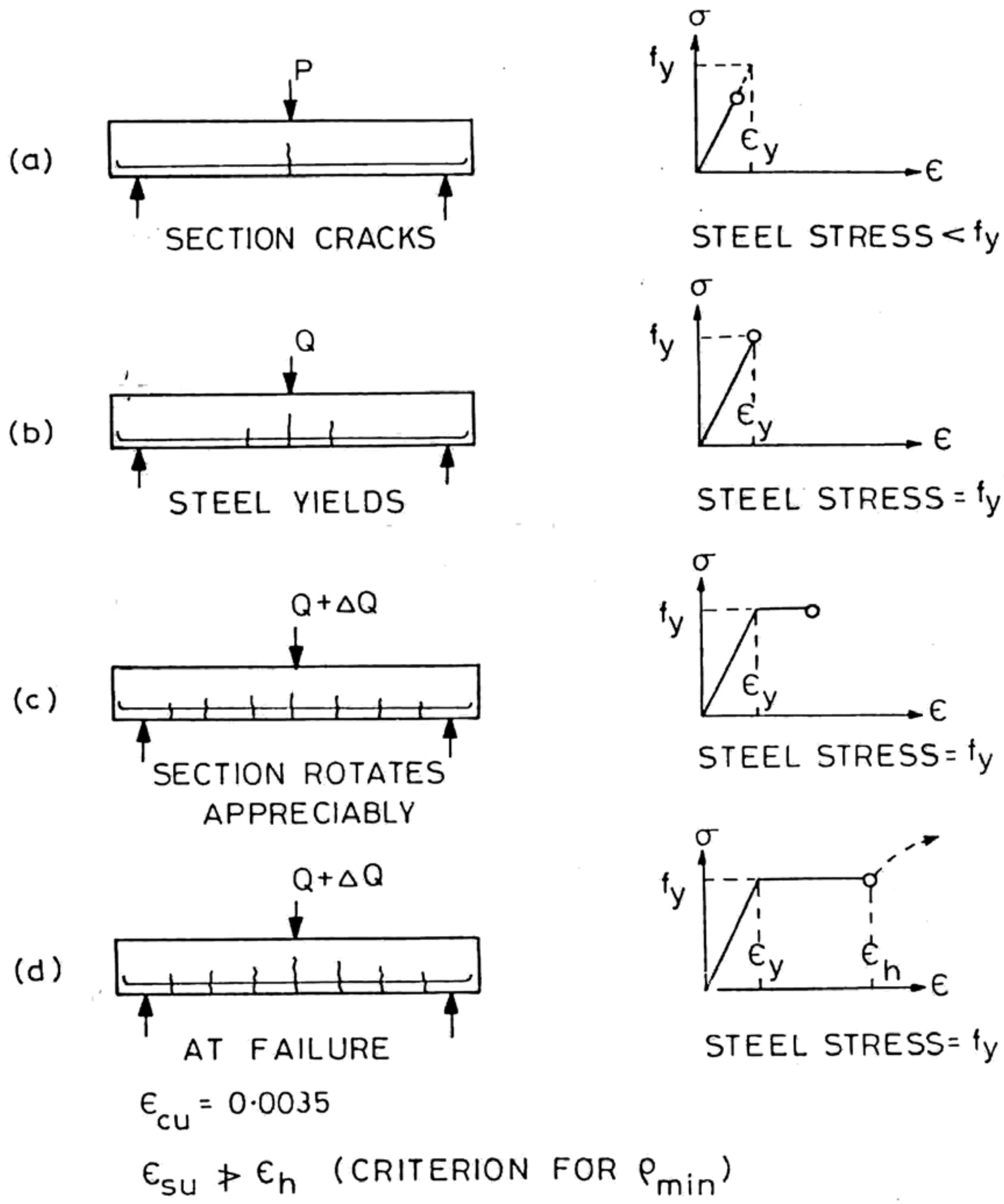


Fig. 4 : Criterion for Determining Minimum Tension Reinforcement-IS: 4326-1976